

2.6 Solving Inequalities Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Solve each linear inequality

a) $x + 3 \leq 5$

$$x \leq 2$$

b) $7x < 4 + 3x$

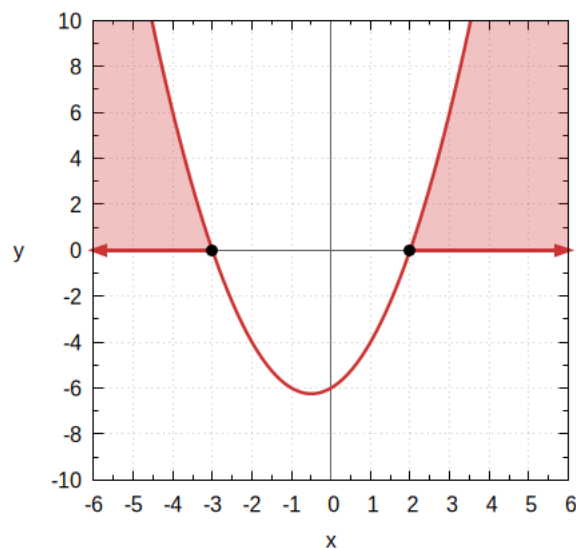
$$4x < 4$$

$$x < 1$$

2) Solve each inequality by graphing

a) $(x + 3)(x - 2) > 0$

Zeros at -3 and 2 . Degree 2, positive leading coefficient.

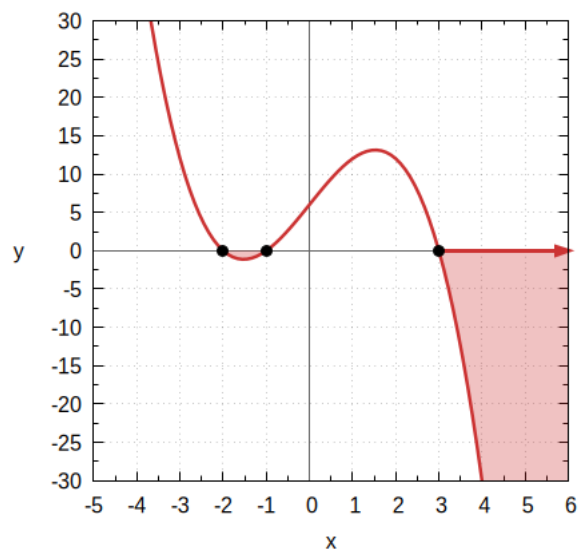


Solution: $x < -3$ or $x > 2$

$$x \in (-\infty, -3) \cup (2, \infty)$$

b) $(x + 2)(3 - x)(x + 1) < 0$

Zeros at -2 , 3 , and -1 . Degree 3, negative leading coefficient (from the $3 - x$ factor).



Solution: $-2 < x < -1$ or $x > 3$

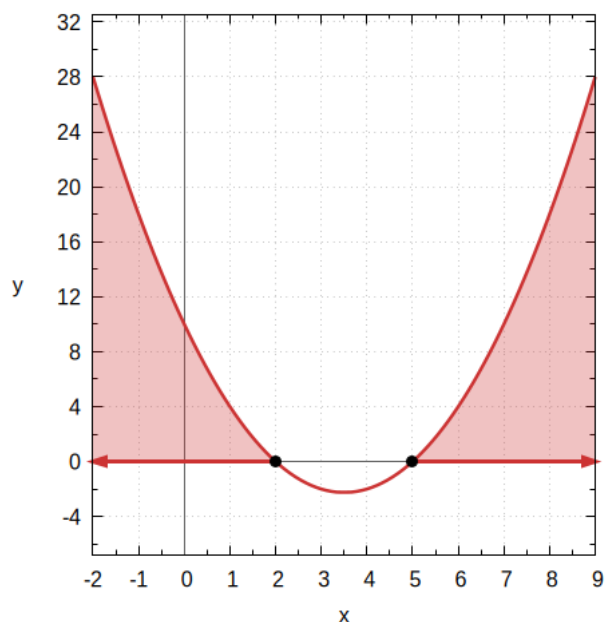
$x \in (-2, -1) \cup (3, \infty)$

3) Solve each of the following polynomial inequalities

a) $x^2 - 7x + 10 \geq 0$

$(x - 2)(x - 5) \geq 0$

	$x < 2$	$2 < x < 5$	$x > 5$
Test point	0	3	6
$x - 2$	-	+	+
$x - 5$	-	-	+
Overall sign	+	-	+



Solution: $x \leq 2$ or $x \geq 5$

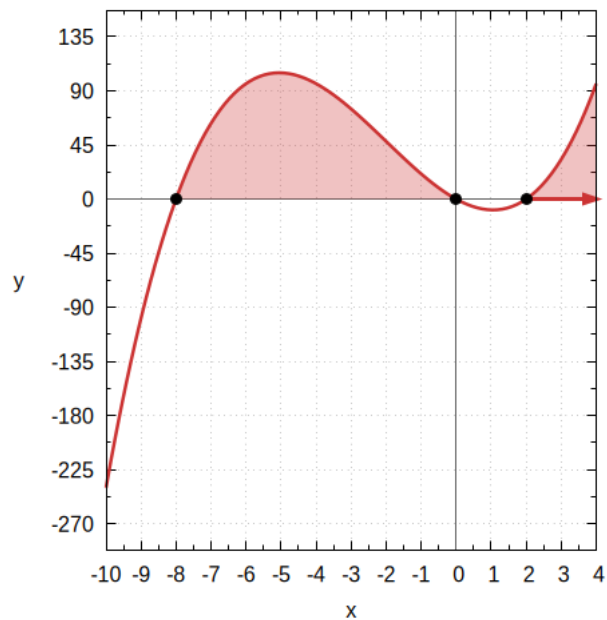
$x \in (-\infty, 2] \cup [5, \infty)$

b) $x^3 + 6x^2 - 16x > 0$

$x(x^2 + 6x - 16) > 0$

$x(x + 8)(x - 2) > 0$

	$x < -8$	$-8 < x < 0$	$0 < x < 2$	$x > 2$
Test point	-10	-1	1	3
x	-	-	+	+
$x + 8$	-	+	+	+
$x - 2$	-	-	-	+
Overall sign	-	+	-	+



Solution: $-8 < x < 0$ or $x > 2$

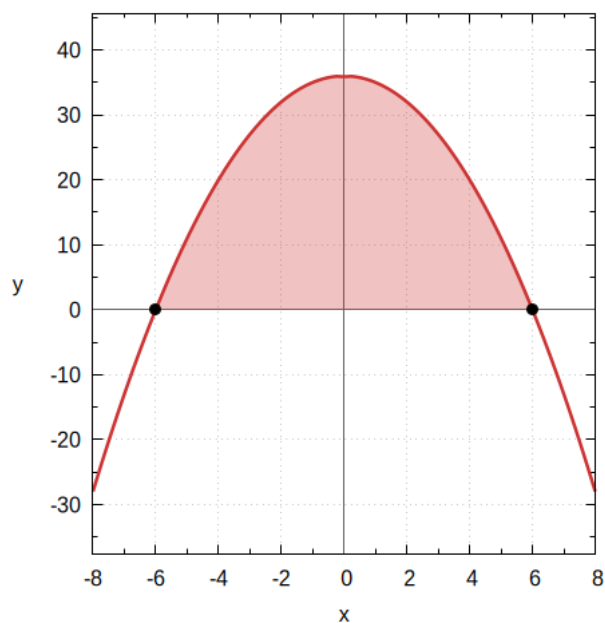
$$x \in (-8, 0) \cup (2, \infty)$$

c) $-x^2 + 36 \geq 0$

$$-1(x^2 - 36) \geq 0$$

$$-1(x - 6)(x + 6) \geq 0$$

	$x < -6$	$-6 < x < 6$	$x > 6$
Test point	-7	0	7
-1	-	-	-
$x - 6$	-	-	+
$x + 6$	-	+	+
Overall sign	-	+	-



Solution: $-6 \leq x \leq 6$

$$x \in [-6, 6]$$

d) $x^4 - 26x^2 + 25 > 0$

$$(x^2 - 25)(x^2 - 1) > 0$$

$$(x - 5)(x + 5)(x - 1)(x + 1) > 0$$

	$x < -5$	$-5 < x < -1$	$-1 < x < 1$	$1 < x < 5$	$x > 5$
Test point	-6	-2	0	2	6
$x - 5$	-	-	-	-	+
$x + 5$	-	+	+	+	+
$x - 1$	-	-	-	+	+
$x + 1$	-	-	+	+	+
Overall sign	+	-	+	-	+

Solution: $x < -5$ or $-1 < x < 1$ or $x > 5$

$$x \in (-\infty, -5) \cup (-1, 1) \cup (5, \infty)$$

e) $x^3 - 3x^2 \geq 25x - 75$

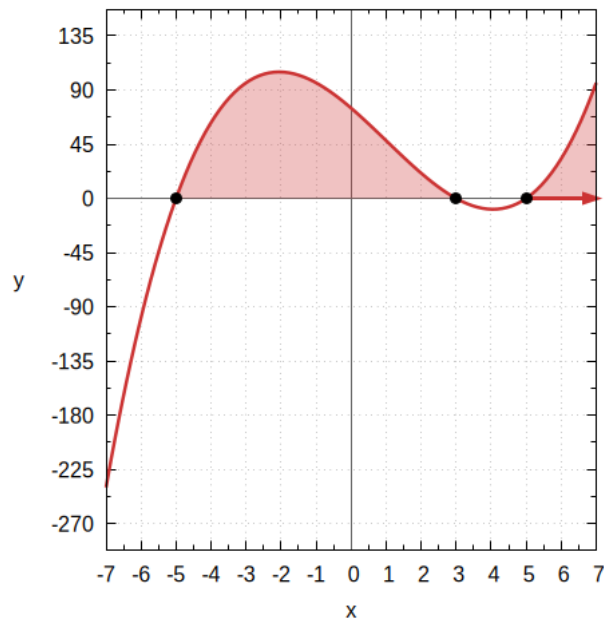
$$x^3 - 3x^2 - 25x + 75 \geq 0$$

$$x^2(x - 3) - 25(x - 3) \geq 0$$

$$(x - 3)(x^2 - 25) \geq 0$$

$$(x - 3)(x - 5)(x + 5) \geq 0$$

	$x < -5$	$-5 < x < 3$	$3 < x < 5$	$x > 5$
Test point	-6	0	4	6
$x - 3$	-	-	+	+
$x - 5$	-	-	-	+
$x + 5$	-	+	+	+
Overall sign	-	+	-	+



Solution: $-5 \leq x \leq 3$ or $x \geq 5$

$$x \in [-5, 3] \cup [5, \infty)$$

f) $-x^3 + 28x + 48 \geq 0$

Multiply both sides by -1 (reverse the inequality):

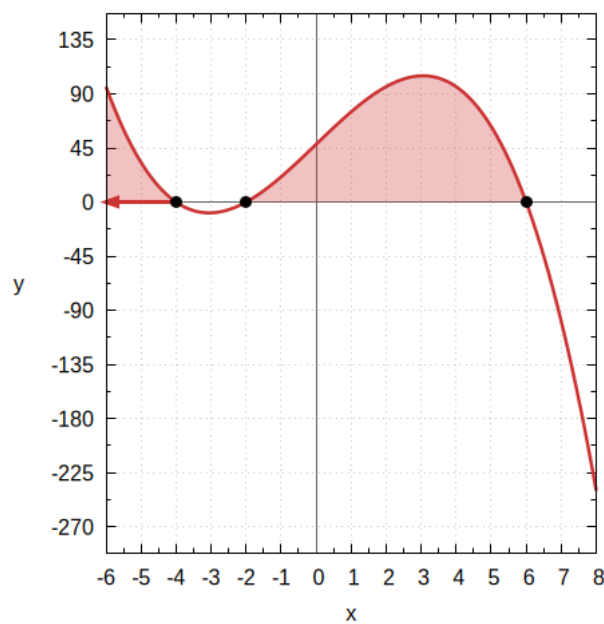
$$x^3 - 28x - 48 \leq 0$$

$$f(-2) = (-2)^3 - 28(-2) - 48 = -8 + 56 - 48 = 0, \text{ so } x + 2 \text{ is a factor.}$$

$$(x + 2)(x^2 - 2x - 24) \leq 0$$

$$(x + 2)(x - 6)(x + 4) \leq 0$$

	$x < -4$	$-4 < x < -2$	$-2 < x < 6$	$x > 6$
Test point	-5	-3	0	7
$x + 2$	-	-	+	+
$x - 6$	-	-	-	+
$x + 4$	-	+	+	+
Overall sign	-	+	-	+



Solution: $x \leq -4$ or $-2 \leq x \leq 6$

$$x \in (-\infty, -4] \cup [-2, 6]$$

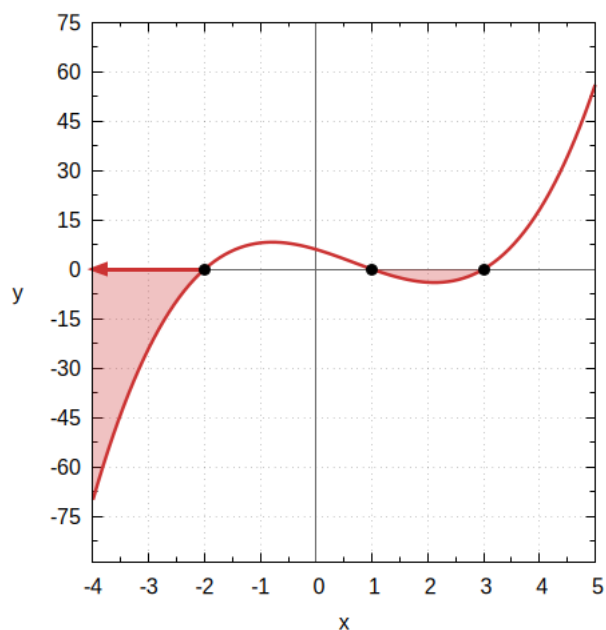
g) $x^3 - 2x^2 - 5x + 6 < 0$

$f(1) = 1 - 2 - 5 + 6 = 0$, so $x - 1$ is a factor.

$$(x - 1)(x^2 - x - 6) < 0$$

$$(x - 1)(x - 3)(x + 2) < 0$$

	$x < -2$	$-2 < x < 1$	$1 < x < 3$	$x > 3$
Test point	-3	0	2	4
$x - 1$	-	-	+	+
$x - 3$	-	-	-	+
$x + 2$	-	+	+	+
Overall sign	-	+	-	+



Solution: $x < -2$ or $1 < x < 3$

$$x \in (-\infty, -2) \cup (1, 3)$$

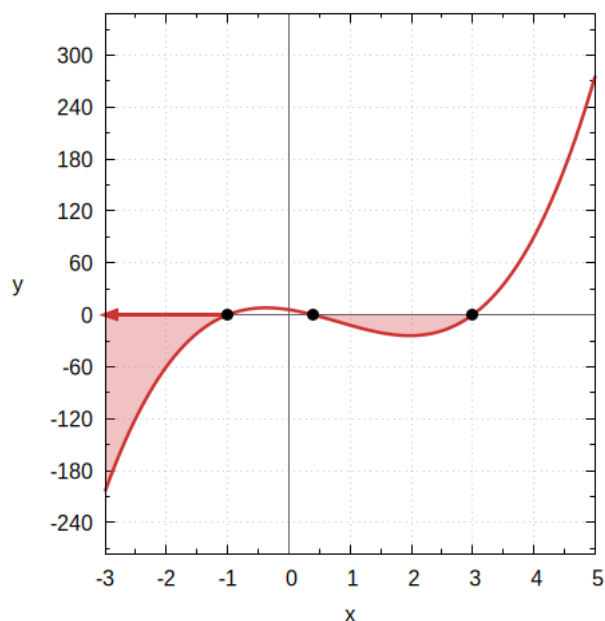
h) $5x^3 - 12x^2 - 11x + 6 \leq 0$

$f(-1) = 5(-1)^3 - 12(-1)^2 - 11(-1) + 6 = -5 - 12 + 11 + 6 = 0$, so $x + 1$ is a factor.

$$(x + 1)(5x^2 - 17x + 6) \leq 0$$

$$(x + 1)(5x - 2)(x - 3) \leq 0$$

	$x < -1$	$-1 < x < 0.4$	$0.4 < x < 3$	$x > 3$
Test point	-2	0	1	4
$x + 1$	-	+	+	+
$5x - 2$	-	-	+	+
$x - 3$	-	-	-	+
Overall sign	-	+	-	+



Solution: $x \leq -1$ or $\frac{2}{5} \leq x \leq 3$

$$x \in (-\infty, -1] \cup \left[\frac{2}{5}, 3\right]$$

4) The price, p , in dollars, of a stock t years after 1999 can be modelled by the function $p(t) = 0.5t^3 - 5.5t^2 + 14t$. When will the stock be more than \$90? You may use technology to help you determine the solution.

$$0.5t^3 - 5.5t^2 + 14t > 90$$

$$0.5t^3 - 5.5t^2 + 14t - 90 > 0$$

$$0.5(t^3 - 11t^2 + 28t - 180) > 0$$

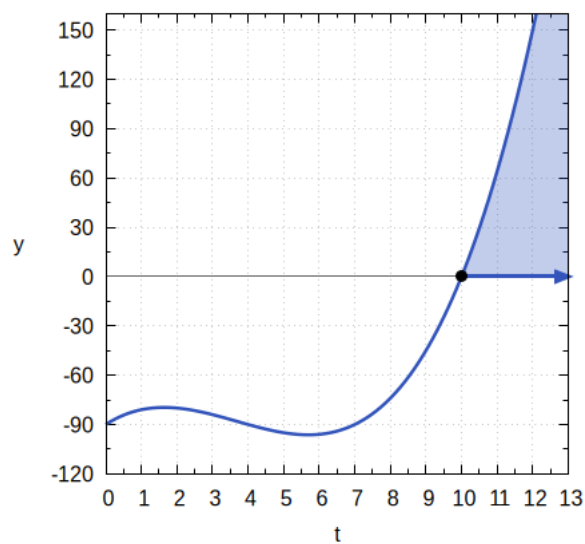
$$t^3 - 11t^2 + 28t - 180 > 0$$

$f(10) = 0$, so $t - 10$ is a factor

$$\begin{array}{r|rrrr} 10 & 1 & -11 & 28 & -180 \\ & & 10 & -10 & 180 \\ \hline & 1 & -1 & 18 & 0 \\ & t^2 & t & \# & R \end{array}$$

$$(t - 10)(t^2 - t + 18) > 0$$

Check the discriminant of $t^2 - t + 18$: $b^2 - 4ac = (-1)^2 - 4(1)(18) = -71$ —no real solutions, so $t^2 - t + 18$ is always positive.



Solution: $t > 10$, $t \in (10, \infty)$

The stock will be more than \$90 after 10 years (2009).