

2.4 Solving Polynomial Equations Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine the solutions of the following polynomials.

a) $(3x + 2)(x + 9)(x - 2) = 0$

$$3x + 2 = 0 \rightarrow x_1 = -\frac{2}{3}$$

$$x + 9 = 0 \rightarrow x_2 = -9$$

$$x - 2 = 0 \rightarrow x_3 = 2$$

Solutions: $\left(-\frac{2}{3}, 0\right)$, $(-9, 0)$, and $(2, 0)$

b) $(x^2 + 1)(x - 4) = 0$

$$x^2 + 1 = 0 \rightarrow x^2 = -1 \text{—no real solutions}$$

$$x - 4 = 0 \rightarrow x = 4$$

Solution: $(4, 0)$

2) Determine the solutions of the following polynomials by factoring. Use the tools you have learned this unit to help you. (remainder theorem, integral zero theorem, division etc.)

a) $x^3 - 4x^2 - 3x + 18 = 0$

Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$$f(-2) = 0; \therefore x + 2 \text{ is a factor}$$

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -3 & 18 \\ & & -2 & 12 & -18 \\ \hline & 1 & -6 & 9 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$(x + 2)(x^2 - 6x + 9) = 0$$

$$(x + 2)(x - 3)^2 = 0$$

$$x_1 = -2, x_2 = 3$$

Solutions: $(-2, 0)$ and $(3, 0)$

b) $x^3 - 3x^2 - 4x + 12 = 0$

Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$f(2) = 0; \therefore x - 2 \text{ is a factor}$$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ & & 2 & -2 & -12 \\ \hline & 1 & -1 & -6 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$(x - 2)(x^2 - x - 6) = 0$$

$$(x - 2)(x - 3)(x + 2) = 0$$

$$x_1 = 2, x_2 = 3, x_3 = -2$$

Solutions: $(2, 0)$, $(3, 0)$, and $(-2, 0)$

c) $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$$f(-1) = 0; \therefore x + 1 \text{ is a factor}$$

$$\begin{array}{r|rrrrrr}
 -1 & 1 & -1 & -11 & 9 & 18 \\
 & & -1 & 2 & 9 & -18 \\
 \hline
 & 1 & -2 & -9 & 18 & 0 \\
 & x^3 & x^2 & x & \# & R
 \end{array}$$

$$(x + 1)(x^3 - 2x^2 - 9x + 18) = 0$$

$$(x + 1)[x^2(x - 2) - 9(x - 2)] = 0$$

$$(x + 1)(x - 2)(x^2 - 9) = 0$$

$$(x + 1)(x - 2)(x - 3)(x + 3) = 0$$

$$x_1 = -1, x_2 = 2, x_3 = 3, x_4 = -3$$

Solutions: $(-1, 0)$, $(2, 0)$, $(3, 0)$, and $(-3, 0)$

d) $x^3 - 64 = 0$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2), a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$(x - 4)(x^2 + 4x + 16) = 0$$

$$x = 4$$

Check $b^2 - 4ac = (4)^2 - 4(1)(16) = -48$ —no real solutions

Solution: $(4, 0)$

e) $2x^3 - 7x^2 + 10x - 5 = 0$

Possible zeros: $\pm 1, \pm \frac{1}{2}, \pm 5, \pm \frac{5}{2}$

$f(1) = 0$; $\therefore x - 1$ is a factor

$$\begin{array}{r|rrrr}
 1 & 2 & -7 & 10 & -5 \\
 & & 2 & -5 & 5 \\
 \hline
 & 2 & -5 & 5 & 0 \\
 & x^2 & x & \# & R
 \end{array}$$

$$(x - 1)(2x^2 - 5x + 5) = 0$$

$$x - 1 = 0 \rightarrow x = 1$$

Check $b^2 - 4ac = (-5)^2 - 4(2)(5) = -15$ —no real solutions

Solution: $(1, 0)$

3) Solve each equation by first factoring the sum or difference of cubes.

a) $x^3 - 8 = 0$

$$(x - 2)(x^2 + 2x + 4) = 0$$

$$x = 2$$

Check $b^2 - 4ac = (2)^2 - 4(1)(4) = -12$ —no real solution

Solution: $(2, 0)$

b) $x^3 + 27 = 0$

$$(x + 3)(x^2 - 3x + 9) = 0$$

$$x = -3$$

Check $b^2 - 4ac = (-3)^2 - 4(1)(9) = -27$ —no real solution

Solution: $(-3, 0)$

4) Solve by factoring

a) $x^3 - 4x^2 - 7x + 10 = 0$

Possible zeros: $\pm 1, \pm 2, \pm 5, \pm 10$

$f(1) = 0$; $\therefore x - 1$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 10 \\ & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \\ x^2 & x & \# & R & \end{array}$$

$$(x - 1)(x^2 - 3x - 10) = 0$$

$$(x - 1)(x - 5)(x + 2) = 0$$

$$x_1 = 1, x_2 = 5, x_3 = -2$$

Solutions: (1, 0), (5, 0), and (-2, 0)

b) $2x^3 - 11x^2 + 12x + 9 = 0$

Possible zeros: $\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$

$f(3) = 0$; $\therefore x - 3$ is a factor

$$\begin{array}{r|rrrr} 3 & 2 & -11 & 12 & 9 \\ & & 6 & -15 & -9 \\ \hline & 2 & -5 & -3 & 0 \\ x^2 & x & \# & R & \end{array}$$

$$(x - 3)(2x^2 - 5x - 3) = 0$$

$$(x - 3)(x - 3)(2x + 1) = 0$$

$$(x - 3)^2(2x + 1) = 0$$

$$x_1 = 3, x_2 = -\frac{1}{2}$$

Solutions: (3, 0) and $(-\frac{1}{2}, 0)$

c) $x^4 - x^3 - 2x - 4 = 0$

Possible zeros: $\pm 1, \pm 2, \pm 4$

$f(-1) = 0$; $\therefore x + 1$ is a factor

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \\ x^3 & x^2 & x & \# & R & \end{array}$$

$$(x + 1)(x^3 - 2x^2 + 2x - 4) = 0$$

$$(x + 1)[x^2(x - 2) + 2(x - 2)] = 0$$

$$(x + 1)(x - 2)(x^2 + 2) = 0$$

$x_1 = -1, x_2 = 2$; $x^2 + 2 = 0$ has no real solution

Solutions: (-1, 0) and (2, 0)