

2.3 Factor Theorem Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine if $x + 3$ is a factor of each polynomial:

a) $x^3 + x^2 - x + 6$

$$\begin{aligned} f(-3) &= (-3)^3 + (-3)^2 - (-3) + 6 \\ &= -27 + 9 + 3 + 6 \\ &= -9 \end{aligned}$$

Not a factor.

b) $2x^3 + 9x^2 + 10x + 3$

$$\begin{aligned} f(-3) &= 2(-3)^3 + 9(-3)^2 + 10(-3) + 3 \\ &= -54 + 81 - 30 + 3 \\ &= 0 \end{aligned}$$

A factor.

c) $x^3 + 27$

$$\begin{aligned} f(-3) &= (-3)^3 + 27 \\ &= 0 \end{aligned}$$

A factor.

2) Find possible factors of the following polynomials using the integral zero theorem. Then, factor the polynomial.

a) $x^3 + 3x^2 - 6x - 8$

Possible factors: $\pm 1, \pm 2, \pm 4, \pm 8$

$f(-1) = 0; \therefore x + 1$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ & & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & 0 \\ x^2 & x & \# & R \end{array}$$

$$x^3 + 3x^2 - 6x - 8 = (x + 1)(x^2 + 2x - 8) = (x + 1)(x + 4)(x - 2)$$

b) $x^3 + 4x^2 - 15x - 18$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$f(-1) = 0; \therefore x + 1$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & 4 & -15 & -18 \\ & & -1 & -3 & 18 \\ \hline & 1 & 3 & -18 & 0 \\ x^2 & x & \# & R \end{array}$$

$$x^3 + 4x^2 - 15x - 18 = (x + 1)(x^2 + 3x - 18) = (x + 1)(x + 6)(x - 3)$$

c) $x^3 - 3x^2 - 10x + 24$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$f(2) = 0; \therefore x - 2$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \\ x^2 & x & \# & R \end{array}$$

$$x^3 - 3x^2 - 10x + 24 = (x - 2)(x^2 - x - 12) = (x - 2)(x - 4)(x + 3)$$

3) Factor by grouping:

a) $x^3 + x^2 - 9x - 9$

$$= (x^3 + x^2) + (-9x - 9)$$

$$= x^2(x + 1) - 9(x + 1)$$

$$= (x + 1)(x^2 - 9)$$

$$= (x + 1)(x - 3)(x + 3)$$

b) $2x^3 - x^2 - 72x + 36$

$$= (2x^3 - x^2) + (-72x + 36)$$

$$= x^2(2x - 1) - 36(2x - 1)$$

$$= (2x - 1)(x^2 - 36)$$

$$= (2x - 1)(x - 6)(x + 6)$$

4) Determine a value of k so that $x + 2$ is a factor of $x^3 - 2kx^2 + 6x - 4$.

$$f(-2) = (-2)^3 - 2k(-2)^2 + 6(-2) - 4$$

$$0 = -8 - 8k - 12 - 4$$

$$0 = -24 - 8k$$

$$24 = -8k$$

$$k = -3$$

5) Find possible factors of the following polynomials using the integral zero theorem. Then, factor the polynomial.

a) $3x^3 + x^2 - 22x - 24$

Possible factors: $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 3, \pm 4, \pm \frac{4}{3}, \pm 6, \pm 8, \pm \frac{8}{3}, \pm 12, \pm 24$

$$f(-2) = 0; \therefore x + 2 \text{ is a factor}$$

$$\begin{array}{r|rrrr} -2 & 3 & 1 & -22 & -24 \\ & & -6 & 10 & 24 \\ \hline & 3 & -5 & -12 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$3x^3 + x^2 - 22x - 24 = (x + 2)(3x^2 - 5x - 12) = (x + 2)(x - 3)(3x + 4)$$

b) $2x^3 - 9x^2 + 10x - 3$

Possible factors: $\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}$

$$f(1) = 0; \therefore x - 1 \text{ is a factor}$$

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 10 & -3 \\ & & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$2x^3 - 9x^2 + 10x - 3 = (x - 1)(2x^2 - 7x + 3) = (x - 1)(x - 3)(2x - 1)$$

c) $6x^3 - 11x^2 - 26x + 15$

Possible factors: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 3, \pm \frac{3}{2}, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{3}, \pm \frac{5}{6}$

$$f(3) = 0; \therefore x - 3 \text{ is a factor}$$

$$\begin{array}{r|rrrr} 3 & 6 & -11 & -26 & 15 \\ & & 18 & 21 & -15 \\ \hline & 6 & 7 & -5 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$6x^3 - 11x^2 - 26x + 15 = (x - 3)(6x^2 + 7x - 5) = (x - 3)(3x + 5)(2x - 1)$$

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d) $4x^3 + 3x^2 - 4x - 3$

Possible factors: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

$f(1) = 0$; $\therefore x - 1$ is a factor

$$\begin{array}{r|rrrr} 1 & 4 & 3 & -4 & -3 \\ & & 4 & 7 & 3 \\ \hline & 4 & 7 & 3 & 0 \\ x^2 & x & \# & R & \end{array}$$

$$4x^3 + 3x^2 - 4x - 3 = (x - 1)(4x^2 + 7x + 3) = (x - 1)(x + 1)(4x + 3)$$

6) Factor each polynomial

a) $2x^3 + 5x^2 - x - 6$

Possible factors: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

$f(1) = 0$; $\therefore x - 1$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -1 & -6 \\ & & 2 & 7 & 6 \\ \hline & 2 & 7 & 6 & 0 \\ x^2 & x & \# & R & \end{array}$$

$$2x^3 + 5x^2 - x - 6 = (x - 1)(2x^2 + 7x + 6) = (x - 1)(x + 2)(2x + 3)$$

b) $4x^3 - 7x - 3$

Possible factors: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

$f(-1) = 0$; $\therefore x + 1$ is a factor

$$\begin{array}{r|rrrr} -1 & 4 & 0 & -7 & -3 \\ & & -4 & 4 & 3 \\ \hline & 4 & -4 & -3 & 0 \\ x^2 & x & \# & R & \end{array}$$

$$4x^3 - 7x - 3 = (x + 1)(4x^2 - 4x - 3) = (x + 1)(2x - 3)(2x + 1)$$

c) $x^4 - 15x^2 - 10x + 24$

Possible factors: $\pm 1, \pm 2, \pm 4$

$f(1) = 0$; $\therefore x - 1$ is a factor

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -15 & -10 & 24 \\ & & 1 & 1 & -14 & -24 \\ \hline & 1 & 1 & -14 & -24 & 0 \\ x^3 & x^2 & x & \# & R & \end{array}$$

$$x^4 - 15x^2 - 10x + 24 = (x - 1)(x^3 + x^2 - 14x - 24)$$

Now further factor $x^3 + x^2 - 14x - 24$:

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$f(-2) = 0$; $\therefore x + 2$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & 0 \\ x^2 & x & \# & R & \end{array}$$

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$$x^4 - 15x^2 - 10x + 24 = (x - 1)(x + 2)(x^2 - x - 12) = (x - 1)(x + 2)(x - 4)(x + 3)$$