

1.3 Factored Form Polynomial Functions Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

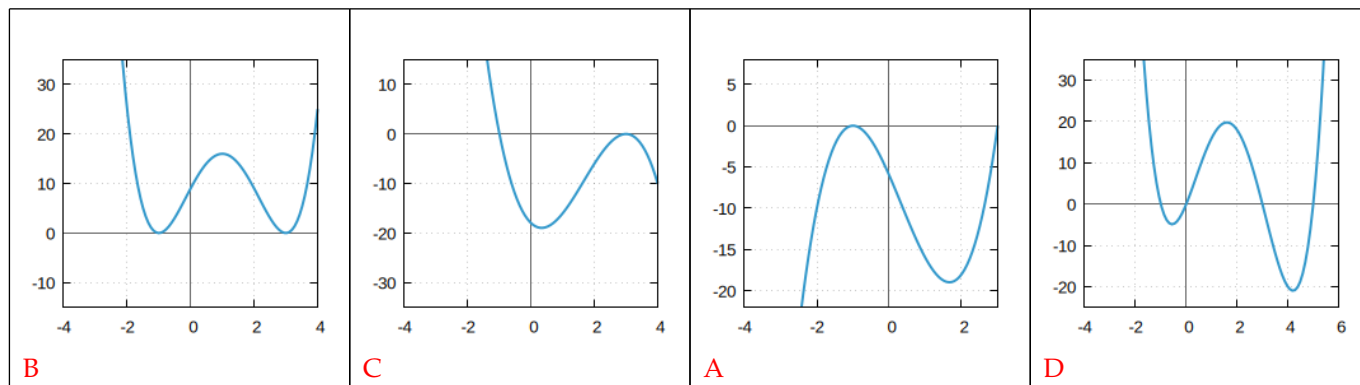
1) Match each equation with the most suitable graph. Write the letter of the equation beneath the matching graph.

A) $f(x) = 2(x + 1)^2(x - 3)$

B) $f(x) = (x + 1)^2(x - 3)^2$

C) $f(x) = -2(x + 1)(x - 3)^2$

D) $f(x) = x(x + 1)(x - 3)(x - 5)$

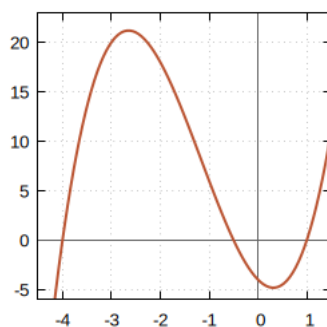


2) Complete the table

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x - 4)(x + 3)(2x - 1)$	3	2	$y \rightarrow -\infty$ as $x \rightarrow -\infty$; $y \rightarrow \infty$ as $x \rightarrow \infty$ (Q3 to Q1)	$(4, 0)$, $(-3, 0)$, $(\frac{1}{2}, 0)$
$g(x) = -2(x + 2)(x - 2)(1 + x)(x - 1)$	4	-2	$y \rightarrow -\infty$ as $x \rightarrow \pm\infty$ (Q3 to Q4)	$(-2, 0)$, $(-1, 0)$, $(1, 0)$, $(2, 0)$
$h(x) = (3x + 2)^2(x - 4)(x + 1)(2x - 3)$	5	18	$y \rightarrow -\infty$ as $x \rightarrow -\infty$; $y \rightarrow \infty$ as $x \rightarrow \infty$ (Q3 to Q1)	$(4, 0)$, $(-1, 0)$, $(-\frac{2}{3}, 0)$, $(\frac{3}{2}, 0)$
$p(x) = -(x + 5)^3(x - 5)^3$	6	-1	$y \rightarrow -\infty$ as $x \rightarrow \pm\infty$ (Q3 to Q4)	$(-5, 0)$, $(5, 0)$

3) For each graph, state...

- the least possible degree and the sign of the leading coefficient
 - the x-intercepts (specify order of zero) and the factors of the function
 - the intervals where the function is positive/negative
- a)



i) degree: 3

leading coefficient: positive

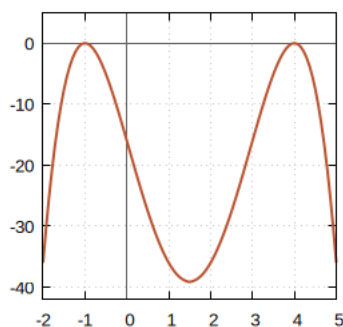
ii) x-intercepts: -4, -0.5, 1

factors: $(x+4)$, $(2x+1)$, $(x-1)$

iii)

Interval	$(-\infty, -4)$	$(-4, -0.5)$	$(-0.5, 1)$	$(1, \infty)$
Sign	-	+	-	+

b)



i) degree: 4

leading coefficient: negative

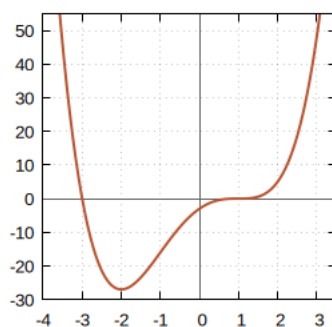
ii) x-intercepts: -1 (order 2), 4 (order 2)

factors: $(x+1)^2$, $(x-4)^2$

iii)

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign	-	-	-

c)



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i) degree: 4

leading coefficient: positive

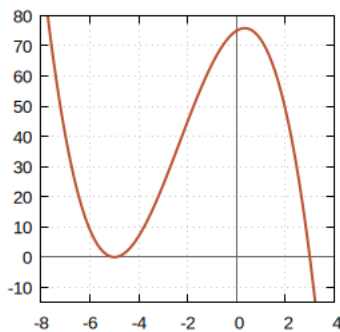
ii) x -intercepts: -3, 1 (order 3)

factors: $(x+3)$, $(x-1)^3$

iii)

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign	+	-	+

d)



i) degree: 3

leading coefficient: negative

ii) x -intercepts: -5 (order 2), 3

factors: $(x+5)^2$, $(x-3)$

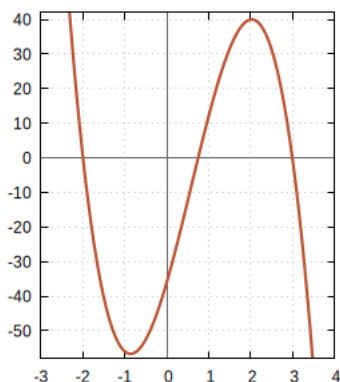
iii)

Interval	$(-\infty, -5)$	$(-5, 3)$	$(3, \infty)$
Sign	+	+	-

4) For each function, complete the chart and sketch a possible graph of the function, labelling key points.

a) $f(x) = -2(x-3)(x+2)(4x-3)$

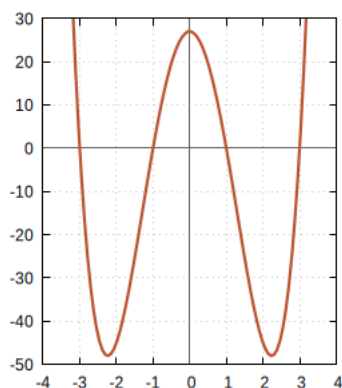
Degree	Leading Coefficient	End Behaviour	x -intercepts	y -intercept
3	-8	$y \rightarrow \infty$ as $x \rightarrow -\infty$; $y \rightarrow -\infty$ as $x \rightarrow \infty$ (Q2 to Q4)	$(3, 0)$, $(-2, 0)$, $(\frac{3}{4}, 0)$	$(0, -36)$



b) $g(x) = (x-1)(x+3)(1+x)(3x-9)$

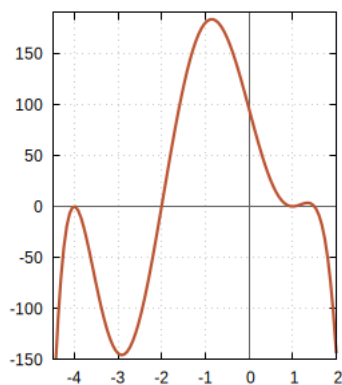
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Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
4	3	$y \rightarrow \infty$ as $x \rightarrow \pm \infty$ (Q2 to Q1)	(1, 0), (-3, 0), (-1, 0), (3, 0)	(0, 27)



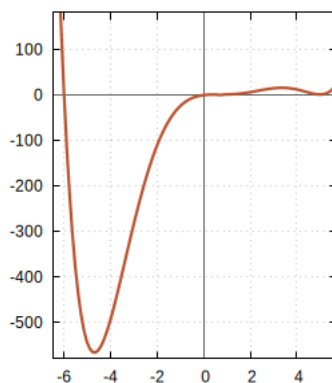
c) $h(x) = -(x+4)^2(x-1)^2(x+2)(2x-3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
6	-2	$y \rightarrow -\infty$ as $x \rightarrow \pm \infty$ (Q3 to Q4)	(-4, 0) order 2, (1, 0) order 2, (-2, 0), (1.5, 0)	(0, 96)



d) $p(x) = 3(x+6)(x-5)^2(3x-2)^3$

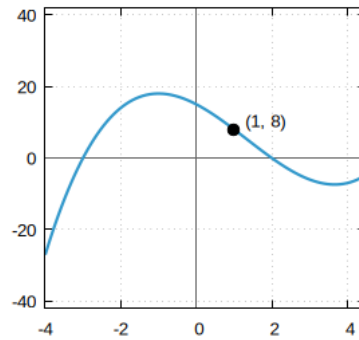
Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
6	81	$y \rightarrow \infty$ as $x \rightarrow \pm \infty$ (Q2 to Q1)	(-6, 0), (5, 0) order 2, ($\frac{2}{3}$, 0) order 3	(0, -3600)



(not to scale)

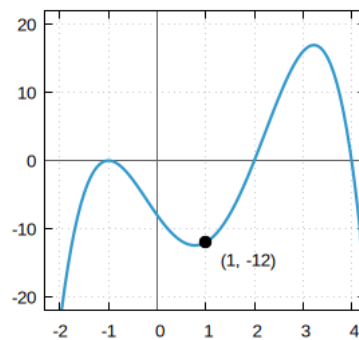
5) Write the equation of each function

a)



Zeros at $-3, 2, 5$, so $f(x) = a(x+3)(x-2)(x-5)$. Substituting $(1, 8)$: $8 = a(4)(-1)(-4) = 16a \Rightarrow a = 0.5$.
 $f(x) = 0.5(x+3)(x-2)(x-5)$

b)



The graph touches (doesn't cross) at $x = -1$, so that's an order-2 zero: $f(x) = a(x+1)^2(x-2)(x-4)$. Substituting $(1, -12)$:
 $-12 = a(4)(-1)(-3) = 12a \Rightarrow a = -1$.

$$f(x) = -(x+1)^2(x-2)(x-4)$$

6) Determine an equation for a quintic function with zeros -1 (order 3) and 3 (order 2) that passes through the point $(-2, 50)$.

$$f(x) = a(x+1)^3(x-3)^2$$

Substituting $(-2, 50)$:

$$50 = a(-1)^3(-5)^2 = a(-1)(25) = -25a \Rightarrow a = -2$$

$$f(x) = -2(x+1)^3(x-3)^2$$

7) Determine the zeros of $f(x) = (2x^2 - x - 1)(x^2 - 3x - 4)$.

Factor each quadratic: $2x^2 - x - 1 = (2x+1)(x-1)$ and $x^2 - 3x - 4 = (x-4)(x+1)$.

$$f(x) = (2x+1)(x-1)(x-4)(x+1)$$

The zeros are $x = 4, 1, -1, \text{ and } -0.5$.