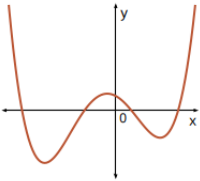
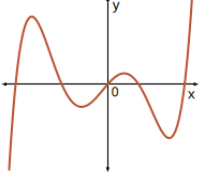
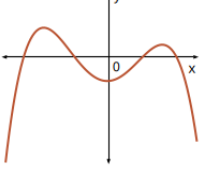
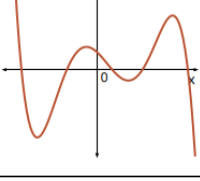
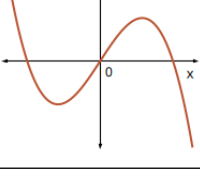


1.2 Characteristics of Polynomial Functions Worksheet with solutions

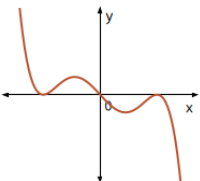
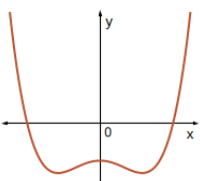
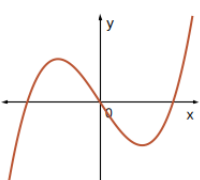
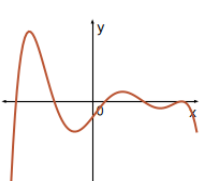
This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	Positive	Even	Q2 to Q1	None	3	4	4
	Positive	Odd	Q3 to Q1	None	4	5	5
	Negative	Even	Q3 to Q4	None	3	4	4
	Negative	Odd	Q2 to Q4	None	4	5	5
	Negative	Odd	Q2 to Q4	Point	2	3	3

2) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	Negative	Odd	Q2 to Q4	Point	4	3	5
	Positive	Even	Q2 to Q1	Line	3	2	4
	Positive	Odd	Q3 to Q1	Point	2	3	3
	Negative	Even	Q3 to Q4	None	5	5	6

3) Complete the following table

Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	Negative	Even	Q3 to Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	Positive	Odd	Q3 to Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	Negative	Odd	Q2 to Q4	2, 0	3, 2, 1
$h(x) = 2x(x - 5)(3x + 2)(4x - 3)$	4	Positive	Even	Q2 to Q1	3, 1	4, 3, 2, 1, 0

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

A) $y = 2x^3 - 4x^2 + 3x + 2$

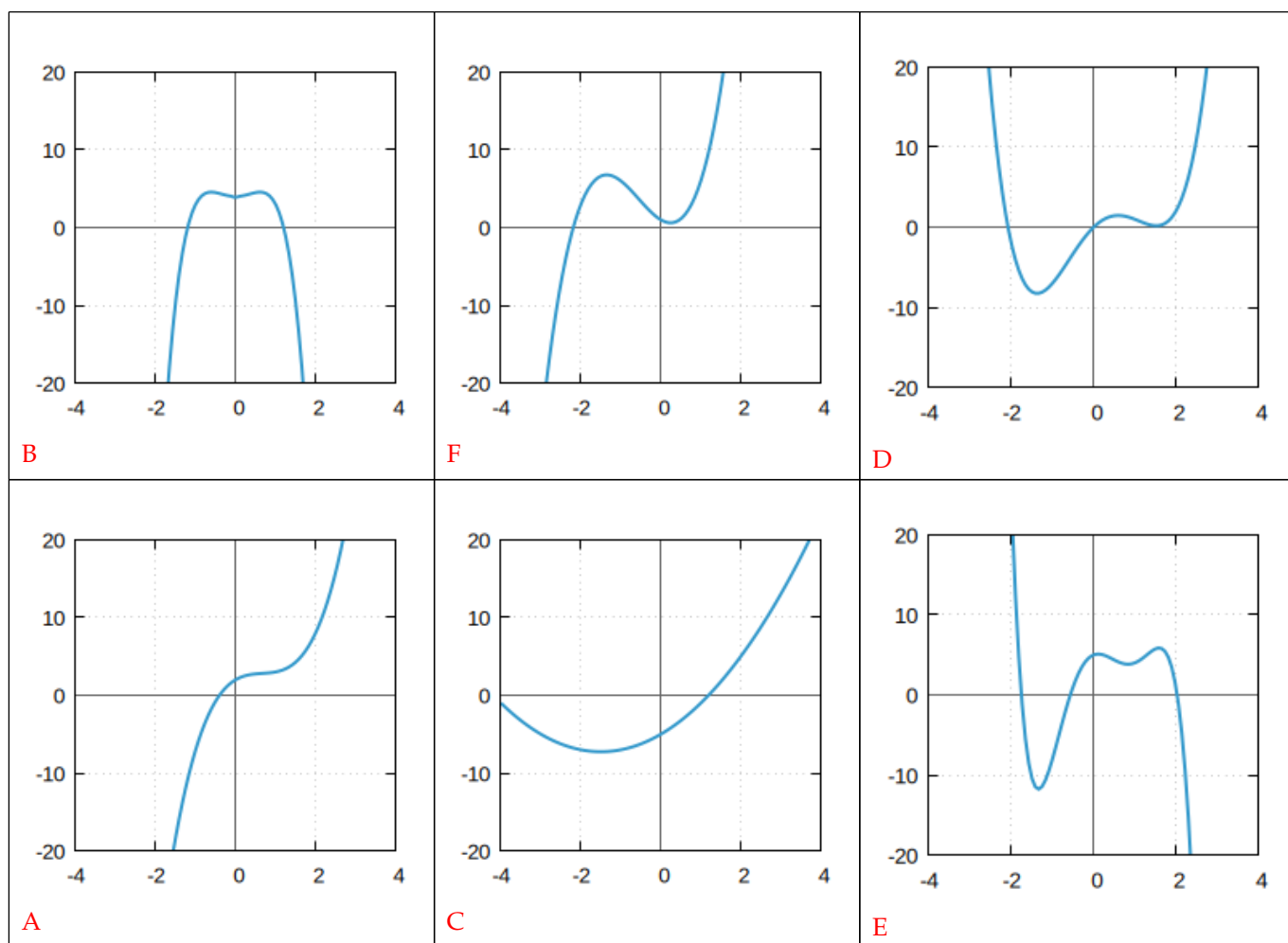
B) $y = -4x^4 + 3x^2 + 4$

C) $y = x^2 + 3x - 5$

D) $y = x^4 - x^3 - 4x^2 + 5x$

E) $y = -2x^5 + 3x^4 + 6x^3 - 10x^2 + 2x + 5$

F) $y = 3x^3 + 5x^2 - 3x + 1$



5) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) second differences = -8

Constant n^{th} differences = $a \cdot n!$. Second differences constant $\Rightarrow$ degree 2. $2a = -8 \Rightarrow a = -4$

b) fourth differences = 24

Fourth differences constant $\Rightarrow$ degree 4. $24a = 24 \Rightarrow a = 1$

6) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y	First differences	Second differences	Third differences
-3	-45	N/A	N/A	N/A
-2	-16	29	N/A	N/A
-1	-3	13	-16	N/A
0	0	3	-10	6
1	-1	-1	-4	6
2	0	1	2	6
3	9	9	8	6
4	32	23	14	6

The third differences are constant (6), so the degree is 3, and $6 = a \cdot 3! = 6a$, so $a = 1$.

b)

x	y	First differences	Second differences	Third differences	Fourth differences
-2	-40	N/A	N/A	N/A	N/A
-1	12	52	N/A	N/A	N/A
0	20	8	-44	N/A	N/A
1	26	6	-2	42	N/A
2	48	22	16	18	-24
3	80	32	10	-6	-24
4	92	12	-20	-30	-24
5	30	-62	-74	-54	-24

The fourth differences are constant (-24), so the degree is 4, and $-24 = a \cdot 4! = 24a$, so $a = -1$.

7) By analyzing the impact of growing economic conditions, a demographer establishes that the predicted population, P , of a town t years from now can be modelled by the function

$$P(t) = 6t^4 - 5t^3 + 200t + 12000$$

a) What is the value of the constant finite differences?

Degree 4, leading coefficient 6, so the constant (fourth) finite differences $= a \cdot n! = 6 \cdot 4! = 6 \times 24 = 144$

b) What is the current population of the town?

$P(0) = 12000$, so the current population is 12 000

c) What will the population of the town be 10 years from now?

$$P(10) = 6(10)^4 - 5(10)^3 + 200(10) + 12000 = 60000 - 5000 + 2000 + 12000 = 69\,000$$