

1.6 Unit 1 Pre-Test Review – Polynomial Functions with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

This review covers all of Unit 1: Polynomial Functions, and is meant to be used to prepare for the unit test.

Section 1: 1.1 Power Functions

1) State the degree and the leading coefficient of each polynomial

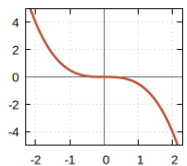
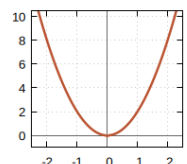
Polynomial	Degree	Leading Coefficient
$y = 2x^3 + 3x - 1$	3	2
$y = 5x - 6$	1	5
$y = x^3 - 2x^2 - 5x^4 + 3$	4	-5
$y = -3x^5 + 2x^3 - x - 1$	5	-3
$y = 21 - 2x + 4x^2 - 6x^3$	3	-6

2) Match each function to its end behaviour

$$y = 3x^7 \quad y = -\frac{1}{2}x^3 \quad y = 2x^4 \quad y = -0.25x^6$$

End Behaviour	Function
Q3 to Q1	$y = 3x^7$
Q2 to Q4	$y = -\frac{1}{2}x^3$
Q2 to Q1	$y = 2x^4$
Q3 to Q4	$y = -0.25x^6$

3) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	Odd	Negative	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$	Point the origin about	Q2 to Q4
	Even	Positive	D: $(-\infty, \infty)$ R: $[0, \infty)$	Line the y-axis about	Q2 to Q1

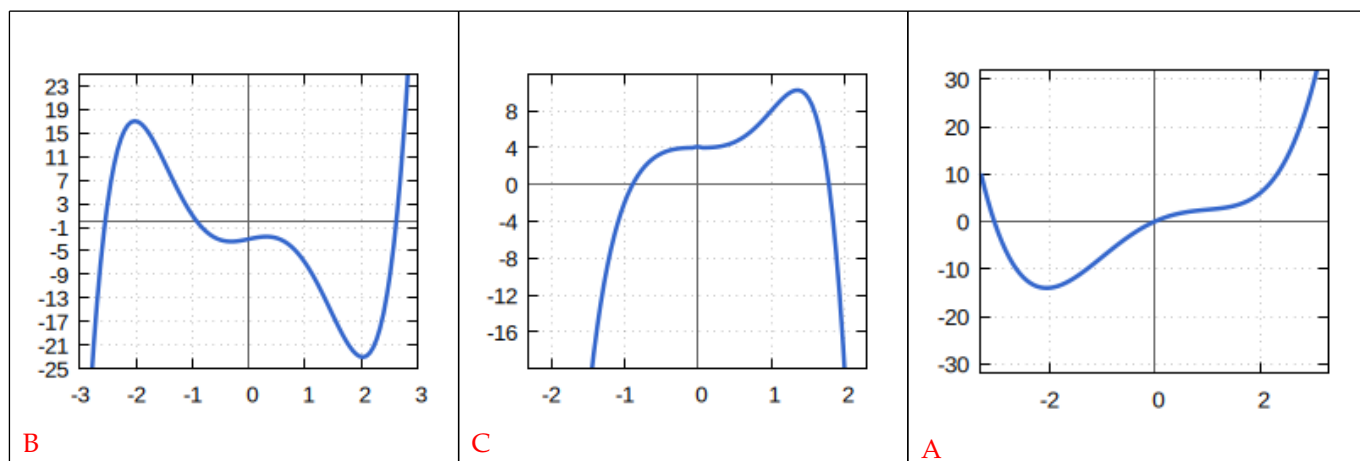
Section 2: 1.2 Characteristics of Polynomial Functions

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

A) $g(x) = 0.5x^4 - 3x^2 + 5x$

B) $h(x) = x^5 - 7x^3 + 2x - 3$

C) $p(x) = -x^6 + 5x^3 + 4$



5) Complete the following table

Equation	Degree	Sign of Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = 6x^3 + 2x$	3	Positive	Q3 to Q1	2, 0	3, 2, 1
$g(x) = -20x^6 - 5x^3 + x^2 - 17$	6	Negative	Q3 to Q4	5, 3, 1	6, 5, 4, 3, 2, 1, 0
$p(x) = 22x^4 - 4x^3 + 3x^2 - 2x + 2$	4	Positive	Q2 to Q1	3, 1	4, 3, 2, 1, 0
$h(x) = -x^5 + x^4 - x^3 + x^2 - x + 1$	5	Negative	Q2 to Q4	4, 2, 0	5, 4, 3, 2, 1

6) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts—Least Possible Degree
	Positive	Even	Q2 to Q1	None	3	4—4
	Negative	Odd	Q2 to Q4	None	4	3—5

7) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) fifth differences = -60

Degree 5. $-60 = a(5!) = 120a \Rightarrow a = -1/2$

b) third differences = 42

Degree 3. $42 = a(3!) = 6a \Rightarrow a = 7$

8) For each function, find the value of the constant finite differences.

a) $g(x) = 0.5x^4 - 3x^2 + 5x$

Finite differences = $a \cdot n! = 0.5(4!) = 0.5(24) = 12$

b) $h(x) = x^5 - 7x^3 + 2x - 3$

Finite differences = $a \cdot n! = 1(5!) = 120$

9) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y	First differences	Second differences	Third differences	
-3	124	N/A	N/A	N/A	
-2	41	-83	N/A	N/A	
-1	8	-33	50	N/A	
0	1	-7	26	-24	
1	-4	-5	2	-24	
2	-31	-27	-22	-24	
3	-104	-73	-46	-24	
4	-247	-143	-70	-24	

The third differences are constant (-24), so the degree is 3, and $-24 = a(3!) = 6a$, so $a = -4$.

b)

x	y	First differences	Second differences	Third differences	Fourth differences	Fifth differences
-2	-229	N/A	N/A	N/A	N/A	N/A
-1	-5	224	N/A	N/A	N/A	N/A
0	3	8	-216	N/A	N/A	N/A
1	-7	-10	-18	198	N/A	N/A
2	-53	-46	-36	-18	-216	N/A
3	-129	-76	-30	6	24	240
4	35	164	240	270	264	240
5	1213	1178	1014	774	504	240

The fifth differences are constant (240), so the degree is 5, and $240 = a(5!) = 120a$, so $a = 2$.

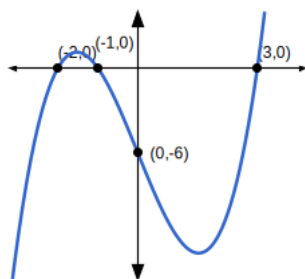
Section 3: 1.3 Factored Form Polynomial Functions

10) For each function, complete the chart and sketch a possible graph of the function labelling key points.

a) $f(x) = (x + 1)(x - 3)(x + 2)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
3	1	Q3 to Q1	$(-1, 0), (3, 0), (-2, 0)$	$(0, -6)$

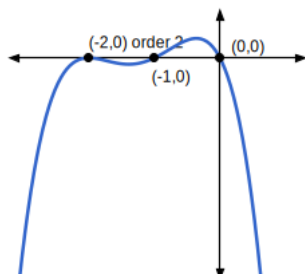
Degree: $(x)(x)(x) = x^3$. Leading coefficient: $(1)(1)(1) = 1$. $f(0) = (0 + 1)(0 - 3)(0 + 2) = (1)(-3)(2) = -6$



b) $g(x) = -x(x+1)(x+2)^2$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
4	-1	Q3 to Q4	(0, 0), (-1, 0), (-2, 0) order 2	(0, 0)

Degree: $(x)(x)(x^2) = x^4$. Leading coefficient: $-1(1)(1)^2 = -1$. $g(0) = -(0)(0+1)(0+2)^2 = -(0)(1)(4) = 0$



c) $h(x) = (x-4)^2(x+3)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
5	1	Q3 to Q1	(4, 0) order 2, (-3, 0) order 3	(0, 432)

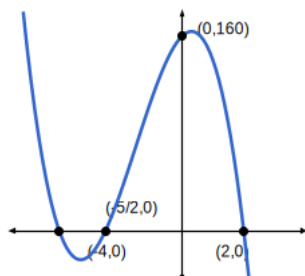
Degree: $(x^2)(x^3) = x^5$. Leading coefficient: $(1)^2(1)^3 = 1$. $h(0) = (0-4)^2(0+3)^3 = (16)(27) = 432$



d) $p(x) = -4(2x+5)(x-2)(x+4)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
3	-8	Q2 to Q4	(-5/2, 0), (2, 0), (-4, 0)	(0, 160)

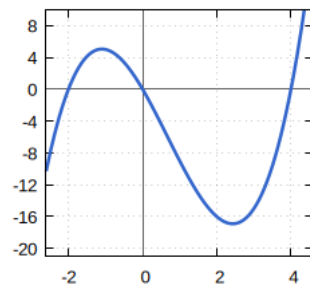
Degree: $(x)(x)(x) = x^3$. Leading coefficient: $-4(2)(1)(1) = -8$. $p(0) = -4[2(0)+5](0-2)(0+4) = -4(5)(-2)(4) = 160$



11) For each graph, state...

- the least possible degree and the sign of the leading coefficient
- the x-intercepts (specify order of zero) and the factors of the function
- the intervals where the function is positive/negative

a)

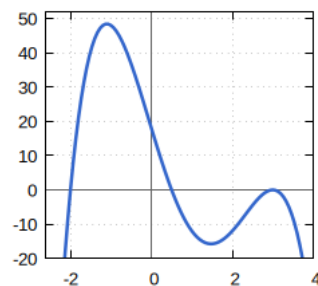


i) degree: 3, leading coefficient: positive

ii) x-intercepts: -2, 0, 4; factors: $(x + 2)$, x , $(x - 4)$

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 4)$	$(4, \infty)$
Sign	-	+	-	+

b)



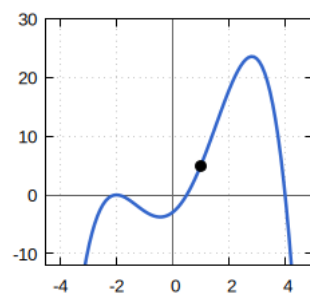
i) degree: 4, leading coefficient: negative

ii) x-intercepts: -2, $1/2$, 3 (order 2); factors: $(x + 2)$, $(2x - 1)$, $(x - 3)^2$

Interval	$(-\infty, -2)$	$(-2, 0.5)$	$(0.5, 3)$	$(3, \infty)$
Sign	-	+	-	-

12) Write the equation of each of the following functions:

a)



The graph has a zero of order 2 at $x = -2$, and simple zeros at $x = 1/2$ and $x = 4$, and passes through the point (1, 5).
 $f(x) = k(x + 2)^2(2x - 1)(x - 4)$

Substituting the point (1, 5):

$$5 = k(1 + 2)^2[2(1) - 1](1 - 4) = k(9)(1)(-3) = -27k \implies k = -\frac{5}{27}$$

$$f(x) = -\frac{5}{27}(x + 2)^2(2x - 1)(x - 4)$$

b) The quartic function has zeros at -3, -1, and 2 (order 2) and passes through the point (1, 4).

$$g(x) = k(x + 3)(x + 1)(x - 2)^2$$

Substituting the point (1, 4):

$$4 = k(1+3)(1+1)(1-2)^2 = k(4)(2)(1) = 8k \implies k = \frac{1}{2}$$

$$g(x) = \frac{1}{2}(x+3)(x+1)(x-2)^2$$

Section 4: 1.4 Transformations of Polynomial Functions

12) Write an equation for the function that results from the given transformations.

a) The function $f(x) = x^4$ is compressed vertically by a factor of $\frac{3}{5}$, stretched horizontally by a factor of 2, reflected horizontally in the y-axis, and translated 1 unit up and 4 units to the left.

$$g(x) = \frac{3}{5} \left[-\frac{1}{2}(x+4) \right]^4 + 1$$

b) The function $f(x) = x^3$ is compressed horizontally by a factor of $\frac{1}{4}$, stretched vertically by a factor of 5, reflected vertically in the x-axis, and translated 2 units to the left and 7 units up.

$$g(x) = -5[4(x+2)]^3 + 7$$

13) Identify the a, k, d and c values and explain what transformation is occurring to the parent function for $g(x) = 2[-4(x+7)]^4 - 1$

$a = 2$; vertical stretch by a factor of 2 ($2y$)

$k = -4$; horizontal reflection and horizontal compression by a factor of $\frac{1}{4}$ ($\frac{x}{-4}$)

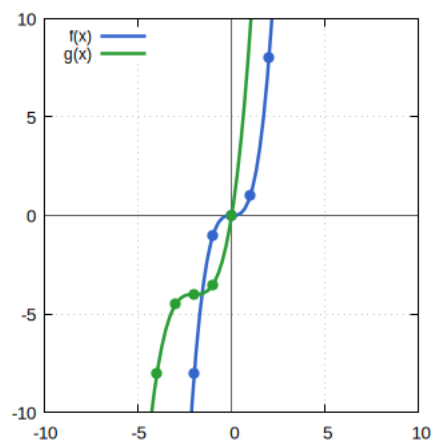
$d = -7$; shift left 7 units ($x-7$)

$c = -1$; shift down 1 unit ($y-1$)

14) For the following questions, use the key points of the parent function to perform transformations. Graph the parent and transformed function. Write the equation of the transformed function.

a) $f(x) = x^3$ $g(x) = \frac{1}{2}f(x+2) - 4$

x (f)	y (f)	x (g)	y (g)
-2	-8	-4	-8
-1	-1	-3	-4.5
0	0	-2	-4
1	1	-1	-3.5
2	8	0	0

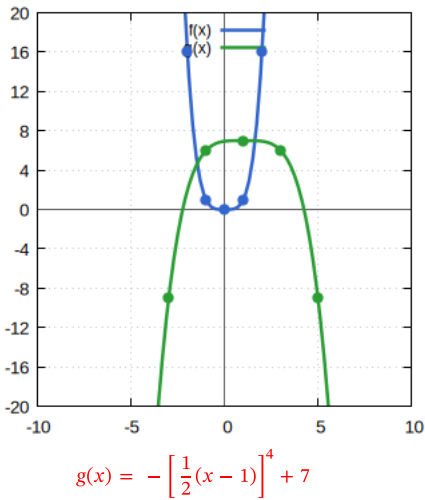


$$g(x) = \frac{1}{2}(x+2)^3 - 4$$

b) $f(x) = x^4$ $g(x) = -f\left[\frac{1}{2}(x-1)\right] + 7$

x (f)	y (f)	x (g)	y (g)
-2	16	-3	-9
-1	1	-1	6
0	0	1	7

1	1	3	6
2	16	5	-9



Section 5: 1.5 Symmetry

5) Circle all that apply for each function

Function/Graph	Symmetry
a) 	No symmetry, Even function, Odd function, Line Symmetry , Point Symmetry
b) 	No symmetry, Even function, Odd function, Line Symmetry, Point Symmetry
c) 	No symmetry, Even function, Odd function, Line Symmetry, Point Symmetry
d) $f(x) = 3x^6 + 2x^2 - 5$	No symmetry, Even function , Odd function, Line Symmetry , Point Symmetry
e) $f(x) = x^3 - 4x^2 + 1$	No symmetry, Even function, Odd function, Line Symmetry, Point Symmetry
f) $f(x) = x^4 + 5x$	No symmetry , Even function, Odd function, Line Symmetry, Point Symmetry

Note: All cubics have point symmetry.

16) Consider the polynomial function $f(x) = -3x^4 + 6x^2 - 10$

a) Show algebraically whether f is even, odd or neither.

$$f(-x) = -3(-x)^4 + 6(-x)^2 - 10 = -3x^4 + 6x^2 - 10 = f(x)$$

Since $f(-x) = f(x)$, it is an even function.

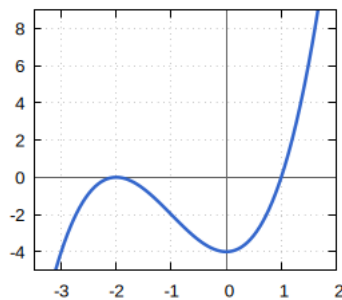
b) For what finite difference will f give a constant value, and what will that constant value be?

It is degree 4, so the fourth differences will be constant. Finite differences = $-3(4!) = -72$

c) What are the maximum and minimum number of zeros the above polynomial could have?

Min zeros = 0, Max zeros = 4. Possible zeros for degree 4 are 4, 3, 2, 1, or 0.

17) Use the given graph to state:



a) x-intercepts

-2 (order 2), and 1

b) number of turning points

2

c) least possible degree

3

d) any symmetry present; even or odd function?

Point symmetry. Not an even or odd function.

e) the intervals where $f(x) < 0$

$f(x) < 0$ when $x < -2$ or $-2 < x < 1$

$f(x) < 0$ when $x \in (-\infty, -2) \cup (-2, 1)$