

1.3 Factored Form Polynomial Functions Lesson

This document was composed and formatted by Annie Bernatchez.

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In this section, you will investigate the relationship between the factored form of a polynomial function and the x -intercepts of the corresponding graph, and you will examine the effect of repeated factors on the graph of a polynomial function.

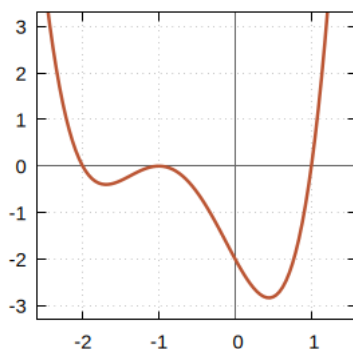
Factored Form Investigation

If we want to graph the polynomial function $f(x) = x^4 + 3x^3 + x^2 - 3x - 2$ accurately, it would be most useful to look at the factored form version of the function:

$$f(x) = (x + 1)^2(x + 2)(x - 1)$$

Let's start by looking at the graph of the function and making connections to the factored form equation.

Graph of $f(x)$:



From the graph, answer the following questions...

- What is the degree of the function?
- What is the sign of the leading coefficient?
- What are the x -intercepts?
- What is the y -intercept?
- The x -intercepts divide the graph into four intervals. Write the intervals in the first row of the table. In the second row, choose a test point within the interval. In the third row, indicate whether the function is positive (above the x -axis) or negative (below the x -axis).

Interval				
Test Point				
Sign of $f(x)$				

- What happens to the sign of $f(x)$ near each x -intercept?

Conclusions from investigation:

The x -intercepts of the graph of the function correspond to the roots (zeros) of the corresponding equation. For example, the function $f(x) = (x - 2)(x + 1)$ has x -intercepts at ____ and _____. These are the roots of the equation $(x - 2)(x + 1) = 0$.

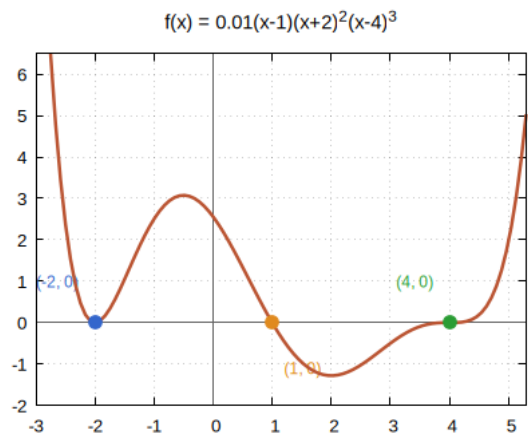
If a polynomial function has a factor $(x - a)$ that is repeated n times, then $x = a$ is a zero of _____ n .

Order – the exponent to which each factor in an algebraic expression is raised.

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For example, the function $f(x) = (x - 3)^2(x - 1)$ has a zero of order _____ at $x = 3$ and a zero of order _____ at $x = 1$.
The graph of a polynomial function changes sign at zeros of _____ order but does not change sign at zeros of _____ order.

Shapes based on order of zero:

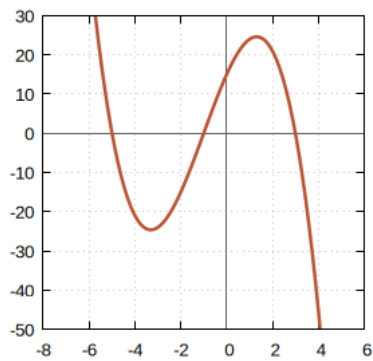


	Order 2	Order 1	Order 3
	(-2, 0) is an x-intercept of order 2. Therefore, it doesn't change sign. "Bounces off" the x-axis. Parabolic shape.	(1, 0) is an x-intercept of order 1. Therefore, it changes sign. "Goes straight through" the x-axis. Linear shape.	(4, 0) is an x-intercept of order 3. Therefore, it changes sign. "S-shape" through the x-axis. Cubic shape.

Example 1: Analyzing Graphs of Polynomial Functions

For each graph,

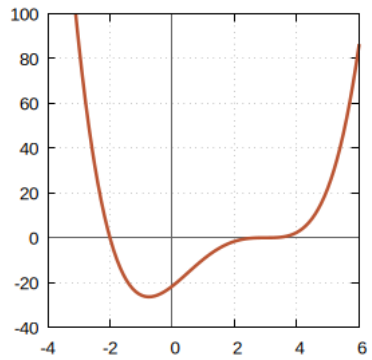
- i) the least possible degree and the sign of the leading coefficient
 - ii) the x -intercepts and the factors of the function
 - iii) the intervals where the function is positive/negative
- a)



- i)
- ii)
- iii)

Interval				
Sign of $f(x)$				

b)



i)

ii)

iii)

Interval			
Sign of $f(x)$			

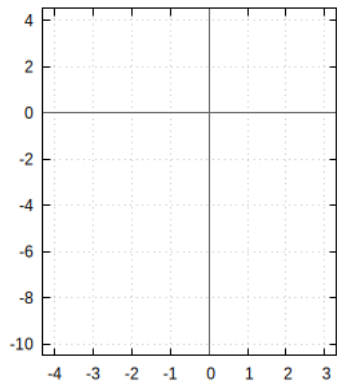
Example 2: Analyze Factored Form Equations to Sketch Graphs

Degree	Leading Coefficient	End Behaviour	x -intercepts	y -intercept
The exponent on x when all factors of x are multiplied together OR Add the exponents on the factors that include an x .	The product of all the x coefficients	Use degree and sign of leading coefficient to determine this	Set each factor equal to zero and solve for x	Set $x = 0$ and solve for y

Sketch a graph of each polynomial function:

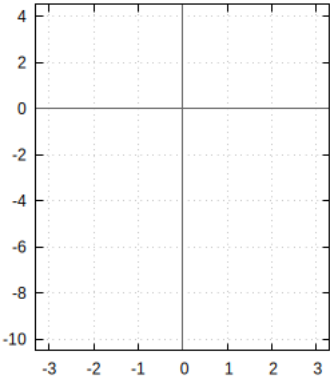
a) $f(x) = (x - 1)(x + 2)(x + 3)$

Degree	Leading Coefficient	End Behaviour	x -intercepts	y -intercept



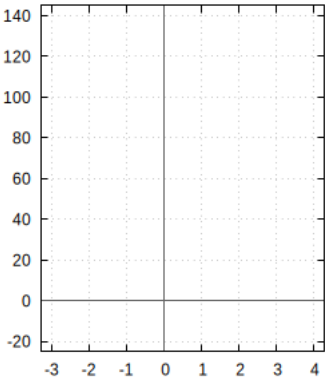
b) $g(x) = -2(x - 1)^2(x + 2)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept



c) $h(x) = -(2x + 1)^3(x - 3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept

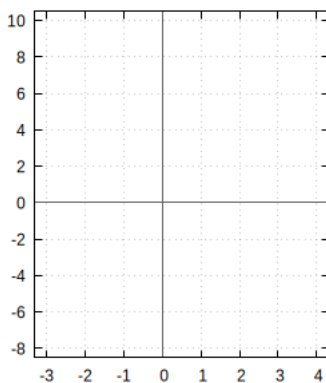


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d) $j(x) = x^4 - 4x^3 + 3x^2$

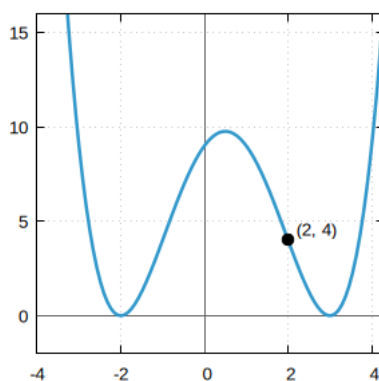
Note: must put into factored form to find x -intercepts

Degree	Leading Coefficient	End Behaviour	x -intercepts	y -intercept



Example 3: Representing the Graph of a Polynomial Function with its Equation

a) Write the equation of the function shown below:



Steps:

1. Write the equation of the family of polynomials using factors created from x -intercepts
2. Substitute the coordinates of another point (x, y) into the equation.
3. Solve for a
4. Write the equation in factored form

b) Find the equation of a polynomial function that is degree 4 with zeros -1 (order 3) and 1 , and with a y -intercept of -2 .