

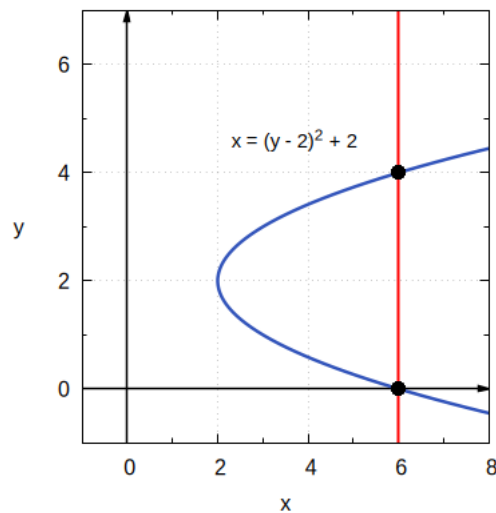
1.1 Polynomial Power Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

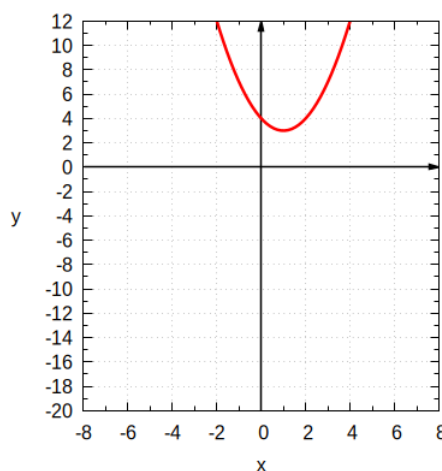
The course material originated from www.jensenmath.ca

Things to Remember About Functions

- A relation is a function if for every x -value there is only 1 corresponding y -value. The graph of a relation represents a function if it passes the **vertical line test**, that is, if a vertical line drawn anywhere along the graph intersects that graph at no more than one point.



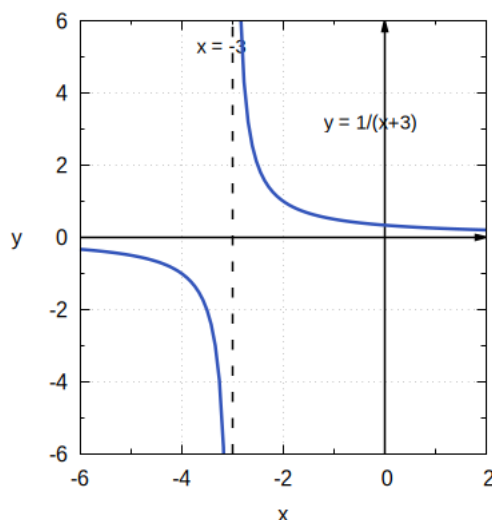
- The **domain** of a function is the complete set of all possible values of the independent variable (x)
 - Set of all possible x -values that will output real y -values
- The **range** of a function is the complete set of all possible resulting values of the dependent variable (y)
 - Set of all possible y -values we get after substituting all possible x -values
- For the function $f(x) = (x - 1)^2 + 3$



- The degree of a function is the highest exponent in the expression
 - $f(x) = 6x^3 - 3x^2 + 4x - 9$ has a degree of **3**.

- An **asymptote** is a line that a curve approaches more and more closely but never touches.

The function $y = \frac{1}{x+3}$ has two asymptotes:



Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq -3$. This is why the vertical line $x = -3$ is an asymptote for this function.

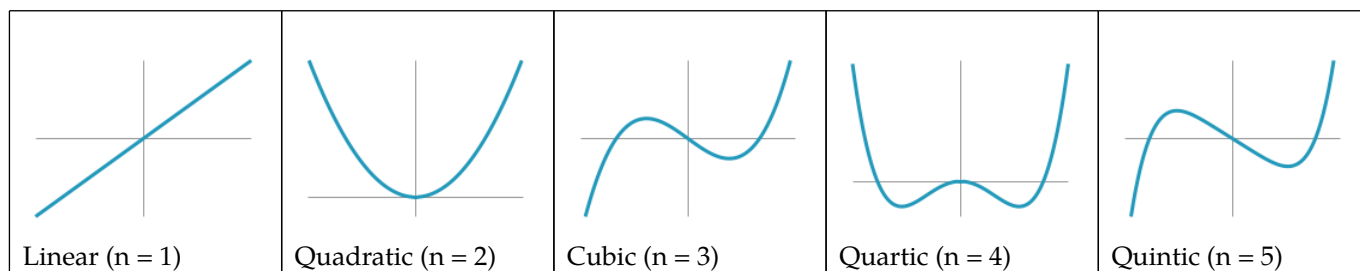
Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will be a horizontal asymptote at $y = 0$.

Polynomial Functions

A polynomial function has the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x^1 + a_0$$

- n is a natural number
- x is a variable
- the **coefficients** $a_0, a_1, \dots, a_n$ are real numbers
- the **degree** of the function is n , the exponent of the greatest power of x
- a_n , the coefficient of the greatest power of x , is the **leading coefficient**
- a_0 , the term without a variable, is the **constant term**
- The domain of a polynomial function is the set of real numbers **(-infinity, infinity)**
- The range of a polynomial function may be all real numbers, or it may have a lower bound or an upper bound (but not both)
- The graph of polynomial functions do not have horizontal or vertical asymptotes
- The graphs of polynomial functions of degree 0 are **horizontal lines**. The shapes of other graphs depends on the degree of the function. Five typical shapes are shown for various degrees:



A **monomial** is the simplest type of polynomial function and has the form:

$$f(x) = ax^n$$

- a is a real number
- x is a variable
- n is a natural number

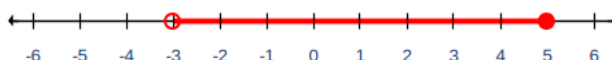
Example 1: Determine which functions are polynomials. State the degree and the leading coefficient of each polynomial function.

a) $g(x) = \sin x$	Not a polynomial – trigonometric function, not a sum of whole-number powers of x .
b) $f(x) = 2x^4$	Polynomial. Degree 4, leading coefficient 2.
c) $y = x^3 - 5x^2 + 6x - 8$	Polynomial. Degree 3, leading coefficient 1.
d) $g(x) = 3^x$	Not a polynomial – exponential function, the variable is in the exponent.

Interval Notation

In this course, you will often describe the features of the graphs of a variety of types of functions in relation to real-number values. Sets of real numbers may be described in a variety of ways:

1. as an inequality $-3 < x \leq 5$
2. interval (or bracket) notation $(-3, 5]$
3. graphically on a number line

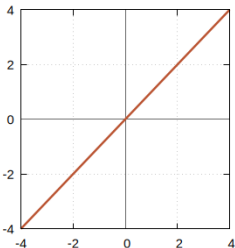
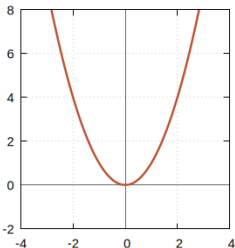
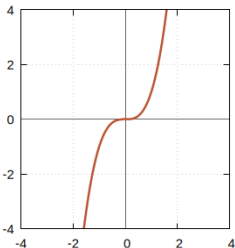
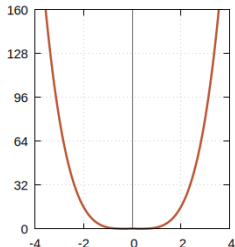
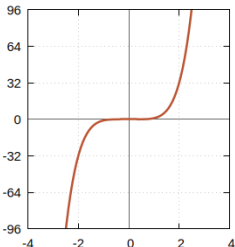
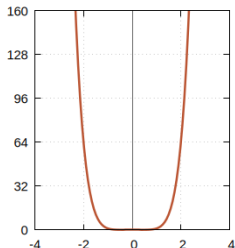


Note:

- Intervals that are infinite are expressed using **infinity** (∞) or **negative infinity** ($-\infty$)
- **Square brackets** $[]$ indicate that the end value is included in the interval
- **Round brackets** $()$ indicate that the end value is NOT included in the interval
- A **round (open)** bracket is always used at infinity and negative infinity

Example 2: Below are the graphs of common polynomial power functions. Use the graph to complete the table.

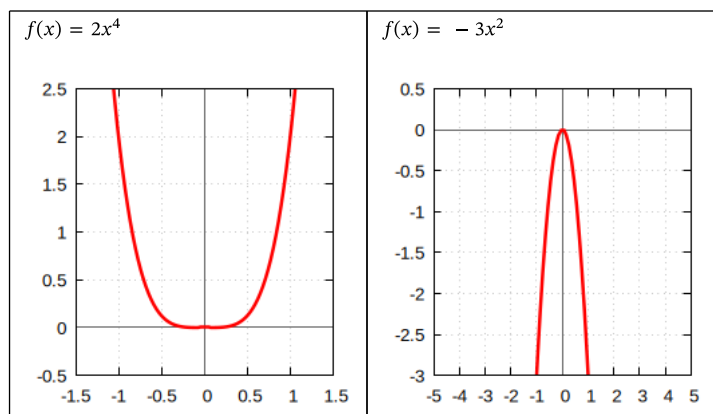
1.1 Polynomial Power Functions Lesson with solutions

Polynomial Power Function	Special Name	Graph	Domain	Range	End Behaviour as $x \rightarrow -\infty$	End Behaviour as $x \rightarrow \infty$
$y = x$	Linear		$(-\infty, \infty)$	$(-\infty, \infty)$	$y \rightarrow -\infty$ (Q3)	$y \rightarrow \infty$ (Q1)
$y = x^2$	Quadratic		$(-\infty, \infty)$	$[0, \infty)$	$y \rightarrow \infty$ (Q2)	$y \rightarrow \infty$ (Q1)
$y = x^3$	Cubic		$(-\infty, \infty)$	$(-\infty, \infty)$	$y \rightarrow -\infty$ (Q3)	$y \rightarrow \infty$ (Q1)
$y = x^4$	Quartic		$(-\infty, \infty)$	$[0, \infty)$	$y \rightarrow \infty$ (Q2)	$y \rightarrow \infty$ (Q1)
$y = x^5$	Quintic		$(-\infty, \infty)$	$(-\infty, \infty)$	$y \rightarrow -\infty$ (Q3)	$y \rightarrow \infty$ (Q1)
$y = x^6$	Sextic		$(-\infty, \infty)$	$[0, \infty)$	$y \rightarrow \infty$ (Q2)	$y \rightarrow \infty$ (Q1)

Key Features of EVEN Degree Polynomial Power Functions

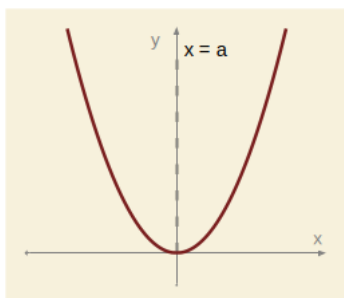
When the leading coefficient (a) is positive	When the leading coefficient (a) is negative
End behaviour: as $x \rightarrow \pm \infty$, $y \rightarrow \infty$ (extends from Q2 to Q1)	End behaviour: as $x \rightarrow \pm \infty$, $y \rightarrow -\infty$ (extends from Q3 to Q4)
Domain: $(-\infty, \infty)$	Domain: $(-\infty, \infty)$
Range: $[0, \infty)$	Range: $(-\infty, 0]$

Example:



Line Symmetry

A graph has line symmetry if there is a vertical line $x = a$ that divides the graph into two parts such that each part is a reflection of the other.

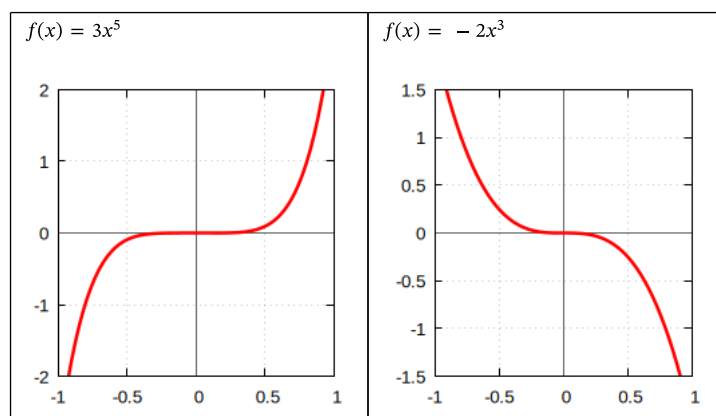


Note: even-degree polynomial power functions (and any function with a single vertex like the parabola shown) have line symmetry about the vertical line through their vertex.

Key Features of ODD Degree Polynomial Power Functions

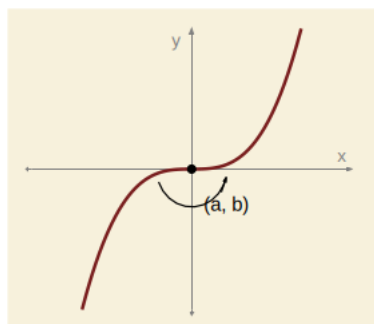
When the leading coefficient (a) is positive	When the leading coefficient (a) is negative
End behaviour: as $x \rightarrow -\infty$, $y \rightarrow -\infty$; as $x \rightarrow \infty$, $y \rightarrow \infty$ (extends from Q3 to Q1)	End behaviour: as $x \rightarrow -\infty$, $y \rightarrow \infty$; as $x \rightarrow \infty$, $y \rightarrow -\infty$ (extends from Q2 to Q4)
Domain: $(-\infty, \infty)$	Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$	Range: $(-\infty, \infty)$

Example:



Point Symmetry

A graph has point symmetry about a point (a, b) if each part of the graph on one side of (a, b) can be rotated 180° to coincide with part of the graph on the other side of (a, b) .



Note: **odd-degree polynomial power functions have point symmetry about the origin $(0, 0)$.**

Example 3: Write each function in the appropriate row of the second column of the table. Give reasons for your choices.

- $y = 2x$, $y = 5x^6$, $y = -3x^2$, $y = x^7$
- $y = -\frac{2}{5}x^9$, $y = -4x^5$, $y = x^{10}$, $y = -0.5x^8$

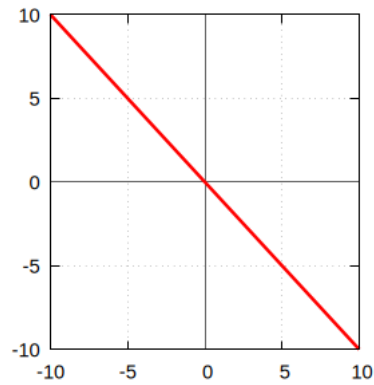
End Behaviour	Functions	Reasons
Q3 to Q1	$y = 2x$, $y = x^7$	Odd degree with a positive leading coefficient: as $x \rightarrow -\infty$, $y \rightarrow -\infty$; as $x \rightarrow \infty$, $y \rightarrow \infty$.
Q2 to Q4	$y = -\frac{2}{5}x^9$, $y = -4x^5$	Odd degree with a negative leading coefficient: as $x \rightarrow -\infty$, $y \rightarrow \infty$; as $x \rightarrow \infty$, $y \rightarrow -\infty$.
Q2 to Q1	$y = 5x^6$, $y = x^{10}$	Even degree with a positive leading coefficient: as $x \rightarrow \pm\infty$, $y \rightarrow \infty$.
Q3 to Q4	$y = -3x^2$, $y = -0.5x^8$	Even degree with a negative leading coefficient: as $x \rightarrow \pm\infty$, $y \rightarrow -\infty$.

Example 4

For each of the following functions

i) State the domain and range ii) Describe the end behavior iii) Identify any symmetry

a) $y = -x$



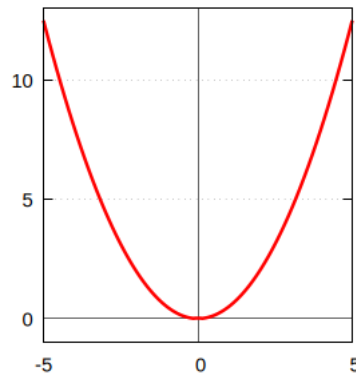
i) Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

ii) As $x \rightarrow -\infty, y \rightarrow \infty$ and as

$x \rightarrow \infty, y \rightarrow -\infty$ The graph extends from Q2 to Q4

iii) Point symmetry about the origin $(0, 0)$ (an odd function)

b) $y = 0.5x^2$



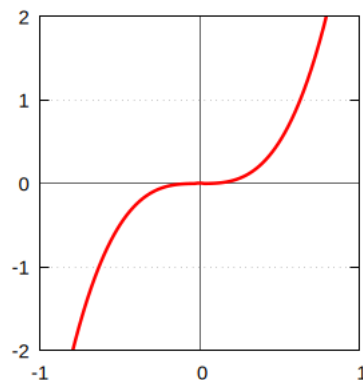
i) Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

ii) As $x \rightarrow -\infty, y \rightarrow \infty$ and as

$x \rightarrow \infty, y \rightarrow \infty$ The graph extends from Q2 to Q1

iii) Line symmetry about the y-axis ($x = 0$)

c) $y = 4x^3$



i) Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

ii) As $x \rightarrow -\infty, y \rightarrow -\infty$ and as

$x \rightarrow \infty, y \rightarrow \infty$ The graph extends from Q3 to Q1

iii) Point symmetry about the origin $(0, 0)$ (an odd function)