

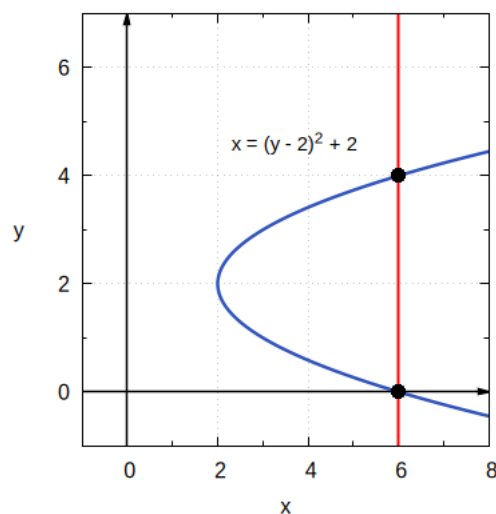
1.1 Polynomial Power Functions Lesson

This document was composed and formatted by Annie Bernatchez.

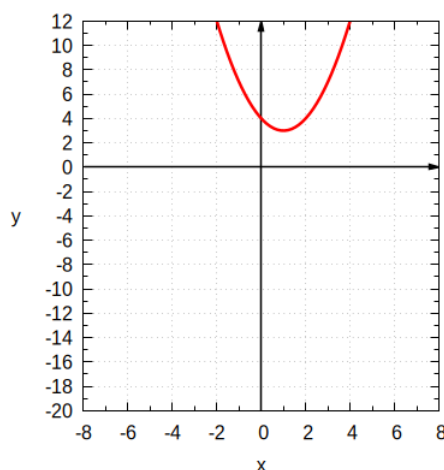
The course material originated from www.jensenmath.ca

Things to Remember About Functions

- A relation is a function if for every x -value there is only 1 corresponding y -value. The graph of a relation represents a function if it passes the _____, that is, if a vertical line drawn anywhere along the graph intersects that graph at no more than one point.



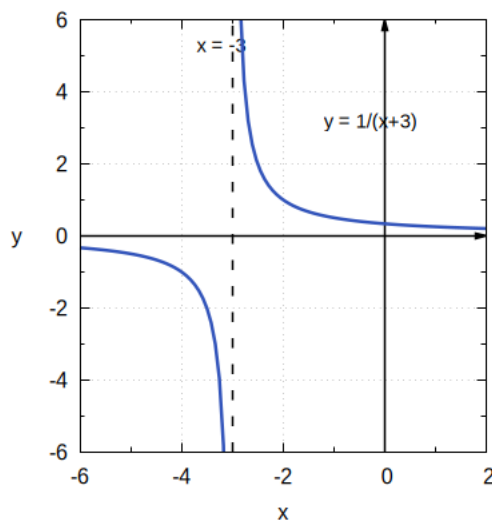
- The _____ of a function is the complete set of all possible values of the independent variable (x)
 - Set of all possible x -values that will output real y -values
- The _____ of a function is the complete set of all possible resulting values of the dependent variable (y)
 - Set of all possible y -values we get after substituting all possible x -values
- For the function $f(x) = (x - 1)^2 + 3$



- The degree of a function is the highest exponent in the expression

- $f(x) = 6x^3 - 3x^2 + 4x - 9$ has a degree of _____.
- An _____ is a line that a curve approaches more and more closely but never touches.

The function $y = \frac{1}{x+3}$ has two asymptotes:



Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq -3$. This is why the vertical line $x = -3$ is an asymptote for this function.

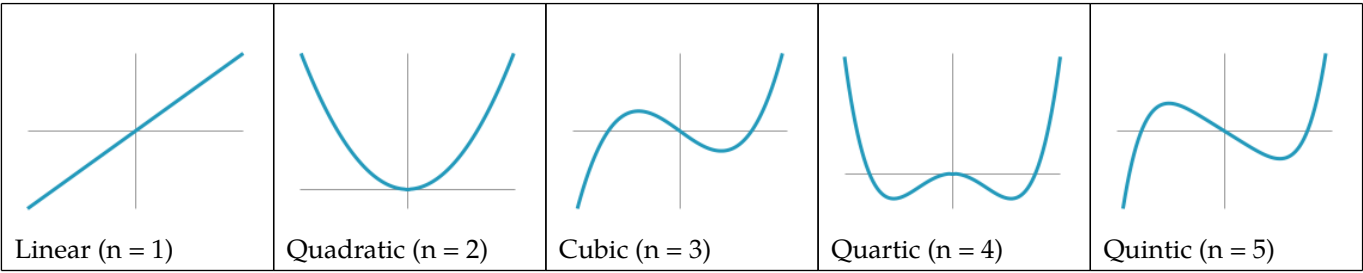
Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will be a horizontal asymptote at $y = 0$.

Polynomial Functions

A **polynomial function** has the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x^1 + a_0$$

- n is a natural number
- x is a variable
- the _____ $a_0, a_1, \dots, a_n$ are real numbers
- the _____ of the function is n , the exponent of the greatest power of x
- a_n , the coefficient of the greatest power of x , is the _____
- a_0 , the term without a variable, is the _____
- The domain of a polynomial function is the set of real numbers _____
- The range of a polynomial function may be all real numbers, or it may have a lower bound or an upper bound (but not both)
- The graph of polynomial functions do not have horizontal or vertical asymptotes
- The graphs of polynomial functions of degree 0 are _____. The shapes of other graphs depends on the degree of the function. Five typical shapes are shown for various degrees:



A _____ is the simplest type of polynomial function and has the form:

$$f(x) = ax^n$$

- a is a real number
- x is a variable
- n is a natural number

Example 1: Determine which functions are polynomials. State the degree and the leading coefficient of each polynomial function.

a) $g(x) = \sin x$	
b) $f(x) = 2x^4$	
c) $y = x^3 - 5x^2 + 6x - 8$	
d) $g(x) = 3^x$	

Interval Notation

In this course, you will often describe the features of the graphs of a variety of types of functions in relation to real-number values. Sets of real numbers may be described in a variety of ways:

1. as an inequality $-3 < x \leq 5$
2. interval (or bracket) notation $(-3, 5]$
3. graphically on a number line

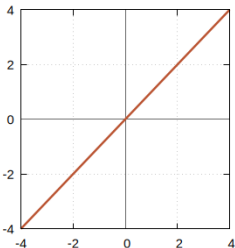
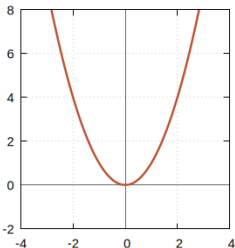
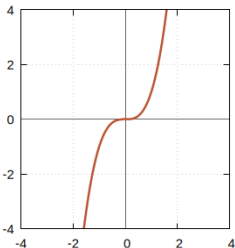
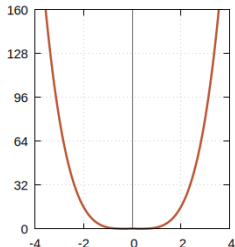
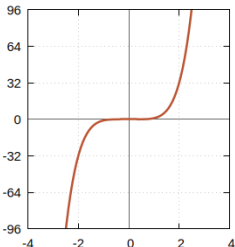
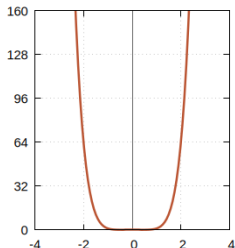


Note:

- Intervals that are infinite are expressed using _____ or _____
- _____ indicate that the end value is included in the interval
- _____ indicate that the end value is NOT included in the interval
- A _____ bracket is always used at infinity and negative infinity

Example 2: Below are the graphs of common polynomial power functions. Use the graph to complete the table.

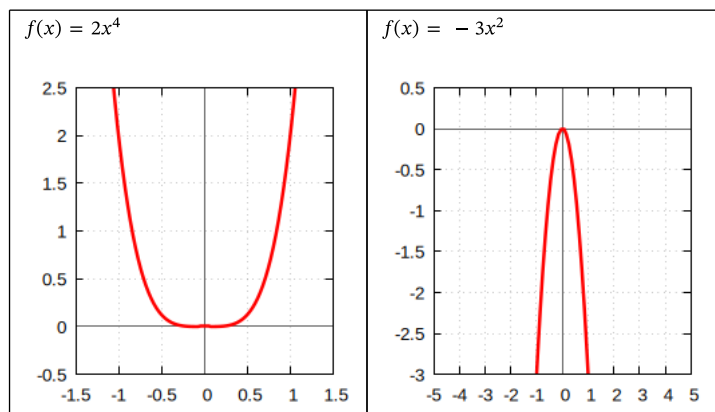
1.1 Polynomial Power Functions Lesson

Polynomial Power Function	Special Name	Graph	Domain	Range	End Behaviour as $x \rightarrow -\infty$	End Behaviour as $x \rightarrow \infty$
$y = x$	Linear					
$y = x^2$	Quadratic					
$y = x^3$	Cubic					
$y = x^4$	Quartic					
$y = x^5$	Quintic					
$y = x^6$	Sextic					

Key Features of EVEN Degree Polynomial Power Functions

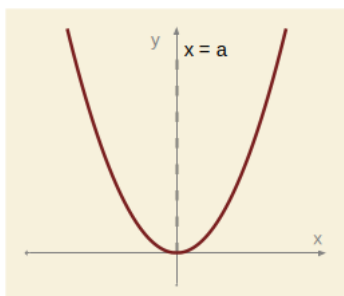
When the leading coefficient (a) is positive	When the leading coefficient (a) is negative
End behaviour:	End behaviour:
Domain:	Domain:
Range:	Range:

Example:



Line Symmetry

A graph has line symmetry if there is a vertical line $x = a$ that divides the graph into two parts such that each part is a reflection of the other.

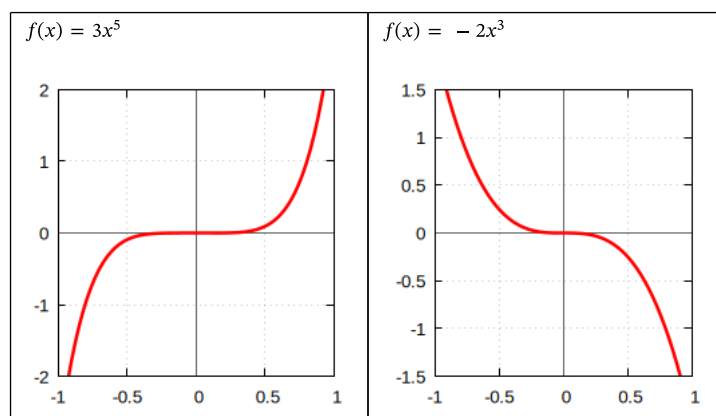


Note:

Key Features of ODD Degree Polynomial Power Functions

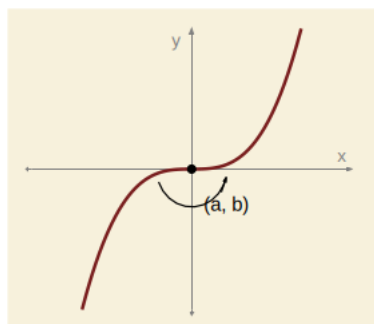
When the leading coefficient (a) is positive	When the leading coefficient (a) is negative
End behaviour:	End behaviour:
Domain:	Domain:
Range:	Range:

Example:



Point Symmetry

A graph has point symmetry about a point (a, b) if each part of the graph on one side of (a, b) can be rotated 180° to coincide with part of the graph on the other side of (a, b) .



Note:

Example 3: Write each function in the appropriate row of the second column of the table. Give reasons for your choices.

- $y = 2x$, $y = 5x^6$, $y = -3x^2$, $y = x^7$
- $y = -\frac{2}{5}x^9$, $y = -4x^5$, $y = x^{10}$, $y = -0.5x^8$

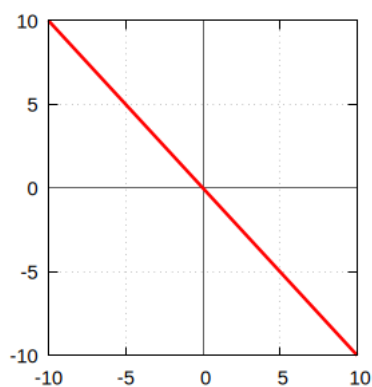
End Behaviour	Functions	Reasons
Q3 to Q1		
Q2 to Q4		
Q2 to Q1		
Q3 to Q4		

Example 4

For each of the following functions

i) State the domain and range ii) Describe the end behavior iii) Identify any symmetry

a) $y = -x$

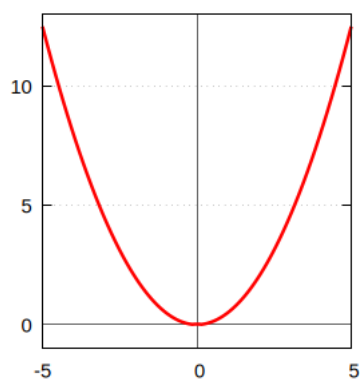


i) Domain: Range:

ii) As _____ and as _____
The graph extends from quadrant _____ to _____

iii)

b) $y = 0.5x^2$

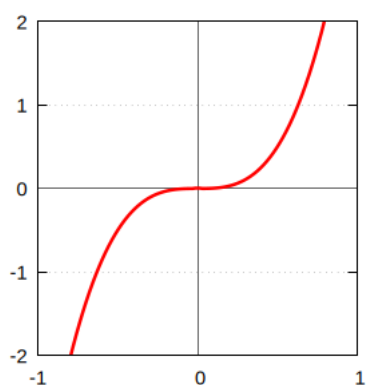


i) Domain: Range:

ii) As _____ and as _____
The graph extends from quadrant _____ to _____

iii)

c) $y = 4x^3$



i) Domain: Range:

ii) As _____ and as _____
The graph extends from quadrant _____ to _____

iii)